Logic
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LOGIC

OR THE

ANALYTIC OF EXPLICIT REASONING

BY

GEORGE H. SMITH


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PREFACE

It is well known to those conversant with the current literature of Logic that recent logical theories diverge widely from the old Logic of Aristotle and the Schoolmen, and no less widely from each other. From this it happens that, under the common name of Logic, we have many doctrines essentially different from each other; and the student who desires to enter upon the study of the subject is thus confronted with the preliminary problem of determining under what name the true Logic is to be found. Nor in this case can he expect much help from his instructors; who, like the rest of the logicians, are hopelessly at a loss. Whether he shall study Logic—whatever may be his wishes and his determination—must therefore be a matter for chance to determine. And, even should he be so lucky as to light on a place where something like Logic is taught, it will probably be taught in so mutilated a form and so mingled with extraneous, and even inconsistent matter, that it will be
impossible for him to understand it or to appreciate its utility. Hence, if the plain truth is to be told, Logic, in the true sense of the term, is no longer taught or learned anywhere; but has become a lost art.

But while the logicians of the day are thus at variance among themselves, there is unfortunately one point in which they agree with each other, and also with Whately and others of the older logicians. This consists in the opinion that Logic is a purely formal science, and as such concerned only with the forms, and not with the matter or content of language or of thought; or, in other words, that it does not deal with what is thought or expressed, but with the forms of the thought or expression only. From this it must follow—if the view be accepted—that Logic, except merely as an improving mental exercise, can be of no practical utility; and this indeed is commonly asserted and always implied in the Logics of the day; which, though essentially different in other respects, agree in this. And from this again it must follow—as on this view was irresistibly argued by Locke, Stewart, Reid, and others—that the subject is unworthy of the serious attention of rational men; which, on the premises assumed, has indeed come to be the verdict of the common sense of mankind. Thus the student is discouraged from
the study of the subject not only by the confusion reigning over it and the almost insurmountable initial difficulty of recognizing the true Logic among so many pretenders, but by the conviction impressed upon him by an irresistible argument and by the practically unanimous teachings of logicians, that Logic cannot be put to any practical use.

The view taken of Logic in this work is different. It is what I conceive to be the ancient and orthodox view, that Logic has to deal with the matter as with the forms of thought and its expression; that it embraces in its scope everything that touches the right use of words, as instruments of reasoning, or, in other words, the whole subject of explicit reasoning or ratiocination; that it is the science fundamental to all others and essential to all who, in the search after truth, would pass beyond the mere evidence of their senses; that, in its educational aspect, it is not only an essential part, but the very foundation of rational education; and finally that, in use, it is indispensable to the rectitude of thought and of life. Hence, of all branches of learning, I believe it to be of the largest practical utility to man, and that all the learning of the day cannot compensate for its loss; and also that its decadence in modern times has been one of the great calamities of mankind. All this I attempt to
establish and to illustrate practically in the following pages; to which I must refer for the complete proofs; but perhaps something towards this end may be effected in advance by explaining briefly how the work came to be written.

In the investigation of Jurisprudence, Politics, and Morality generally—to which my studies have been principally devoted—two important facts were forced on my attention, that seem to establish my present thesis:

(1) The first of these was that the prevailing errors in the theory of Politics, Sociology, and Morality, and the Moral Sciences, or Science of Human Nature, generally, have their sources, almost always, in merely logical fallacies, and may be readily refuted by the application of familiar logical principles; all of which will be practically illustrated in treating of the fallacies. Here, then, I think, we have a practical proof of the indispensable utility of Logic, and the consequent refutation of the error that it deals only with the forms of thought or expression. For it is known to all logicians that the most serious and pernicious of the recognized fallacies are those that relate to the matter expressed in language, and are therefore called the material fallacies; which by logicians generally are admitted into Logic, but, as it were, on sufferance only.
(2) The second fact I learned was that, though it is impracticable to refute such errors otherwise than by the application of logical principles, yet owing to the logical decadence of the age, and the general disuse of Logic, this mode of refutation is unavailable. Hence under existing conditions, there is no practical means of stemming the tide of moral and political heresy with which, with increasing violence, mankind is being afflicted; and from this it follows, as a necessary inference, that the first step towards reform of doctrine, or life, in any direction, must be a revival of the study and use of Logic. My work therefore is the result of a profound realization of this practical necessity, and of the imperative demand thus resulting. Nor—however interesting the theory of Logic may have been to me—have I ever lost sight of what I conceive to be the most important aspect of the subject, namely, its supreme practical utility.

Generally, the object of the work is to vindicate, as against modern innovations, the old or traditional Logic. This constitutes a perfectly definite body of doctrine, rivalling in accuracy and in demonstrative force the Geometry of Euclid. Nor are there wanting treatises in which its theory and application are, on the whole, well explained,—as, e. g., notably Whately's work; which, notwithstanding some
manifest defects, still remains, not only the best, but the only elementary exposition of Logic, in the English language, that can be recommended to the student. But there are many reasons why a mere reproduction of the older works would be inadequate for our present occasions, to some of which I will briefly advert.

The first of these relates to the error, already considered, that prevailed with many of the old logicians, as with the new, that Logic is concerned only with the forms, and not with the matter of thought, or its expression. For, though this defect was supplied by the old logicians,—at the expense of their consistency,—by their admirable exposition of the doctrines of Definition and of Classification and Division and of the Term generally, and of the Material or so-called Non-logical Fallacies, yet their theory of Logic remained incomplete, and Logic was thus mutilated of some of its most vital parts.

Again, the searching investigation to which the old Logic has been subjected by modern logicians, though its general effect has been to vindicate its substantial truth and to re-establish it on a broader and firmer basis, has yet resulted in several additions to logical doctrine, to which it is essential that the attention of the student should be directed. Hence, while one
of the principal objects of this work is to vindicate the truth and the supreme utility of Logic as anciently conceived, it is also contemplated to supply the radical defect I have alluded to, and, at the same time, to incorporate with the old Logic the approved results of modern research; some of which are of great importance.

It remains to add a few words as to the method and style with which the subject of the work is treated. Logic is admittedly a demonstrative or apodictic doctrine, and should therefore be treated by the method appropriate to subjects of that nature. This consists in the accurate formulation of our premises, and in reasoning rigorously from them, as in geometry. But this method demands the use of a style altogether different from that in common use; which may be called the popular or rhetorical. For it is the peculiar characteristic of the logical style that it must be accurate or aphoristic, i.e., that it must express the exact truth without any admixture of error. For the same truth holds good in ratiocination, as in nature generally, that hybrids are unprolific; and hence the slightest admixture of error in our premises will render them altogether useless for logical inference. Our method will therefore demand the exact analysis of the terms we use and the formal statement of our propositions; which to the general reader is
distasteful. For while the logical style admits, and even requires, great brevity of expression,—so that, in general, volumes of ordinary disquisition may, by means of it, be compressed into a brief space,—yet it demands a degree of attention and independent thought that only a few highly trained or exceptionally gifted minds are willing to give, or perhaps without great exertion are capable of giving. But this is nevertheless essential to the fruitful study of Logic, as of apodictic science generally. There is no royal road to Logic any more than to Geometry.

The best type of this style is found in the Mathematics, and especially in the writings of Euclid and the geometers, whose style and method I have sought to emulate,—with what success remains to be judged. I trust, however, I may, without vanity, say of the result, with Hobbes, that while "there is nothing I distrust more than my elocution, nevertheless I am confident, excepting the mischances of the press, it is not obscure."

GEORGE H. SMITH.

LOS ANGELES, February 26, 1900.
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LOGIC, OR THE ANALYTIC OF EXPLICIT REASONING

INTRODUCTION

OF THE FUNCTION OF LOGIC

§ 1. THE THEORY OF KNOWLEDGE, A DEPARTMENT OF THE THEORY OF OPINION.
—The problem of the origin and nature of knowledge has occupied the attention of the philosophers for something over twenty-five centuries without much progress toward solution. This perhaps results from the fact that the problem itself is but part of a larger problem that should be first considered; for knowledge is but a species of opinion, which may be either true or false. Hence the inquiry as to the origin and nature of opinion must be the first in order of investigation. Nor until this investigation has been made will we be prepared to determine the specific characteristics
by which true knowledge is differentiated from opinion in general.

§ 2. Knowledge but Verified Opinion. —Men generally confound this distinction, and regard all their settled opinions or beliefs as knowledge. This is not merely false, but absurd; for not only do the opinions of men differ, but the opinions of the same man are often inconsistent and contradictory; and some, it is clear, must be false. And this is apparent also from the nature and generation of our opinions. For, in general, these come to us not from any conscious process, but naturally and spontaneously and from many sources, as, e.g., from testimony, from authority, from inaccurate observation or careless reasoning, and even largely from mere prejudice or bias. Hence, familiar to us as our opinions are, their origin in general is as unknown to us as were anciently the sources of the Nile; nor have we any just notion of the grounds on which they rest, or of the nature and justice of their demands on our belief. Hence, until some means of verifying our opinions be found and applied, we can have no assurance of their rectitude. The first step in Science or Philosophy must, therefore, be to distinguish between verified and unverified opinions. The former constitutes true knowledge or science; the latter—though it is in
fact the stuff out of which most of the current philosophy is woven—has no just pretension to the name.

§ 3. The Sources of Opinion Distinguished.—With regard to the source of our opinions, we must distinguish between those derived from our own experience and those derived from the experience of others; of which those derived from the common experience of mankind are the most extensive and important. The last have come to us by means of language, which may therefore be said to be their source; nor could they otherwise have been transmitted to us. The former constitute—comparatively speaking—but a small and insignificant part of the sources of the mass of our opinions. For the greater part of what we know, or think we know, is not original with us, but has come to us from others by or from language. The distinction, therefore, is, not between opinions derived from experience and opinions not so derived,—for it may be said all opinions that are true, or rather that we know to be true, are derived ultimately from experience,1—but in the manner of their derivation; the one class being those opinions derived by us, each from his own experience, the other, those derived not directly from our own,

1 The distinction made in the text is of fundamental importance. The necessity of a constant resort to experience as the
but from the experience of others from or through language.

§ 4. Of Language as a Record of Human Thought.—Of the two classes of opinions, the latter is infinitely the more extensive in scope and important in character; for all that men have seen or thought or felt has been expressed, and is thus preserved to us in language; which thus constitutes, as it were, the record of the results of all human experience and reason. Here, therefore, is to be found the principal source of our opinions, verified and unverified—that is to say, not only of our opinions generally, but of our knowledge or science. But, regarding language as a record and source of opinion, we must distinguish between the forms in which opinion is embodied in it. These forms may be described, with sufficient accuracy for our purposes, as consisting in terms, propositions, and syllogisms. But of these the syllogism in its end and effect is but the reduction of two ultimate source of our knowledge cannot be too strongly insisted upon. But to construe this proposition as referring to each man's individual experience is to fall into an error of the kind called by Bacon "Idols of the Den"; and thus to fall under the reproach of Heraclitus "that men search for knowledge in lesser worlds, and not in the greater or common world," i. e., the great world of the common notions of mankind, derived from the universal experience and embodied in the common language. (Nov. Org., bk. i., aph. xliii.)
propositions to one, and, in this connection, is of interest to us merely as exhibiting one of the modes in which opinion is formed. It will be sufficient, therefore, to distinguish the term and the proposition as the two forms in which opinions, or the elements of opinions, are embodied. But the proposition is itself of two kinds, differing essentially in nature. In the one—if not an inference—it is simply the statement of a relation intuitively perceived to exist between two terms or names, that is to say, between the notions or concepts denoted by them,—as, e. g., where we say, "Bodies are affected by gravity," or "Two islands cannot be contiguous," or "Fishes live in the sea," or "Man is rational"; in the other, it is a statement of a relation between terms, not intuitively perceived—or logically inferred—but assumed to be true from testimony or otherwise,—as, e. g., where we say, "Brutus was one of the murderers of Cæsar," or "Hannibal was conquered by the Romans." The former—in accordance with the definitions used throughout this work—will be called a judgment; the latter, an assumption. In the former case the truth of the proposition is involved in the meanings of the terms,—i. e., in the nature of the concepts or notions denoted by them; and this is true also of all inferences, or propositions inferred from judgments. So that with
relation to all such propositions, whether intuitively perceived or inferred, the original sources of opinion are the notions or concepts in which they are involved. We may therefore distinguish, as the two sources of opinion afforded us by language, (1) the notions or concepts expressed in terms, and (2) assumptions, or assumed propositions.

With the truth of the latter, or the evidence on which they rest for credence, Logic is not concerned; nor is it concerned with them in any way, except as premises from which to argue; or to reject them as such, if they can be shown by logical processes to be false. But where such propositions are justified by experience, and come thus to be generally received, the result universally, or almost universally, is the generation of a new notion, — *i. e.*, the notion of the relation perceived between its terms; which is either expressed in a new term or added to the content or meaning of an existing term; and this, indeed, to the extent it is attainable, is the end of science, and, in a perfect language, — were such attainable, — would be the general result. Thus the general progress of human thought consists largely in the conversion of propositions into terms or names denoting the relations expressed in them; and hence, generally, in terms are contained many propositions, as, *e. g.*, in "gravity," "justice,"
etc.—in the former of which is contained a large part of Physical Science, and in the latter nearly the whole theory of the State. In this way the stock of the common notions of mankind is continuously accumulated, until it may be said that the great part of all that has been achieved in thought by men is expressed or implied in terms or names. Here, therefore, are to be found the principal sources of opinion; and, compared with these, opinions embodied in propositions that cannot be, or have not been, reduced to single notions are limited in extent, and of secondary importance. And this is especially true with regard to the Moral Sciences; under which name I include all the various branches of the science of human nature; for in these sciences it is impossible to conceive of any rudimentary notion or thought that has not, in the long history of man, been conceived by the human mind and embodied in terms. With reference, therefore, to all that has been achieved in science or in popular thought, the sources of all our opinions, verified and unverified,—that is to say, of all our knowledge or supposed knowledge,—are to be sought in language, and, principally, in the notions expressed in terms or names; and consequently, with reference to

1 If the reader will thoroughly apprehend this proposition, he will find in it the key, not only to Logic, but to all Phil-
knowledge or supposed knowledge of this kind, our method must consist in the study of language.

§ 5. Received Opinion Distinguished from True Knowledge.—Our opinions, however, are derived from this source in two ways, which must be distinguished: namely, by tradition,—by which our opinions are delivered to us ready made in the form of propositions,—and by reasoning upon the notions embodied in terms. For the thought contained in language is embodied in two ways, namely, *explicitly*, in the form of propositions, and *implicitly*, in terms; and of propositions,—as we have seen,—many are but explicit statements of what is implied in the notions of knowledge. The *elements* of knowledge, so far as already achieved, we repeat, are the notions or concepts incarnate in terms; and these must always constitute the principal source of our knowledge; for, in comparison with the knowledge thus expressed or implied, the original contributions of the most gifted of men to the common stock must be inconsiderable. Nor can any such contribution to the knowledge of mankind be regarded as completely achieved until embodied in definite terms; and hence the formation of such terms, or, what is the same, of the notions embodied in them, must be regarded as the end of scientific discovery. There is, therefore, nothing paradoxical in the assertion of Condillac that "Science is but language well made." Hence, to repeat what has been said, it is to the common stock of notions thus gradually accumulated by mankind and permanently secured by expression in terms, that we must resort as the principal source of all knowledge or science. See Appendix A.
expressed in terms, as, *e. g.*, in the proposition, "All bodies are affected by gravity," etc. With reference to these, though they may be true, their mere reception cannot be said to constitute knowledge; but—in the proper sense of the terms—we can know them only when we have reasoned them out for ourselves from the primary notions in which they are involved; as, *e. g.*, in the Mathematics, where we cannot be said to have mastered a theorem until we are able to work it out from the premises by the exertion of our own powers unassisted by memory. With reference to all that has been achieved in thought, therefore, our method in the pursuit of knowledge must begin with the apprehension of the notions already formed by men and embodied in terms; and this involves the testing of those notions for ourselves by comparing them with the realities to which they are supposed to correspond.

§ 6. The Physical and Mathematical, Distinguished from the Moral Sciences. — These observations apply equally to the Physical and Mathematical as to the Moral Sciences; but there are differences, partly essential and partly accidental, between the two classes of sciences, which must be adverted to:

(1) In the Physical Sciences and in the

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1 See Appendix B.
Mathematics, technical terms expressing accurately the concepts or notions involved are exclusively used, but in the Moral Sciences it is otherwise; for there the notions developed by the experience and reasoning of mankind—which must always constitute the principal source of our knowledge—are in general loosely and inaccurately expressed, and the same vocal sign, or vocable, is commonly used to denote many different notions more or less nearly related; nor, with reference to these, does the term in general express the notion accurately. Hence the necessity of definition, which is at once the fundamental and the most difficult of the logical processes. But in the Physical Sciences the notion is always accurately defined by the thing itself; and so in the Mathematics, though highly abstract, our notions are always clearly defined. Thus in these sciences the logical processes are so simple that it is impossible to err, unless by inadvertence, and all errors are quickly corrected; and hence a technical knowledge of Logic is but little needed.¹ But in the Moral Sciences it is different, for here the difficulty of defining our terms is

¹Hence, from disuse of the more difficult of the logical processes, a man in the former case, may be a competent naturalist without being much of a reasoning creature; and in the latter, a great mathematician and yet a child in the practical affairs of life, individual and social.
great, and often insuperable, and hence, in the prosecution of these sciences, Logic must always be an indispensable instrument.

(2) To a certain extent this difference between the two classes of sciences is an essential one, and cannot be altogether removed. But to a large degree the Moral Sciences are susceptible of apodictic treatment, and by such treatment may be indefinitely assimilated in nature to what are commonly called — though not exclusively entitled to the name — the Exact Sciences; for a large part of the Moral Sciences, including nearly all the fundamental principles upon which they rest, are purely apodictic. For, though it is commonly supposed there is an essential difference between Mathematical and what is called Moral Reasoning, this is not true; all ratiocination (not fallacious) is essentially of the same character and equally conclusive.¹

(3) Hence it may be observed as a corollary,

¹This is much insisted upon by Locke: “Confident I am,” he says, “that if men would, in the same method, and with the same indifferency, search after moral, as they do after mathematical truths, they would find them to have a stronger connection, one with another, and a more necessary consequence from our clear and distinct ideas, and to come nearer a perfect demonstration than is commonly supposed” (Essay, bk. iv., chap. iii., 20). “By what steps we are to proceed . . . is to be learned in the school of the mathematicians, who, from very plain and easy beginnings, by gentle degrees,
the principal task before us, with reference to the Moral Sciences, is to reduce them as far as possible to apodictic or scientific form. This, under present conditions, will still leave an immense field of investigation in which we must resort directly to experience, and especially to experience as embodied in history and statistics; but until all that is susceptible of being so reduced is reduced to scientific form, no progress can be made in dealing with matters depending upon experience.

(4) With regard to the Physical Sciences another difference is to be noted, namely, between what has been achieved and the discovery of new facts; with reference to which the instrument of discovery is mainly experiment and observation, or, as it is commonly called, the Inductive Method. In this respect these differ from the Moral Sciences, where, though the same method must always be used, its function is confined chiefly to the process of definition.¹

and a continued chain of reasonings, proceed to the discovery and demonstration of truths that appear at first sight beyond human capacity" (*Id.*, bk. iv., chap. xii., 7, 8). "This gave me confidence to advance the conjecture which I suggest, Chap. iii., viz., that Morality is capable of demonstration as well as Mathematics."

¹ The nature of Logic, and of the relation of the Inductive Method to Logic, is thus precisely expressed by Bacon:

"The syllogism consists of propositions, propositions of
§ 7. Of the Modes in which Opinion is Generated.—With reference to results achieved and embodied in language, and to our opinions generally, the process by which our notions or concepts are derived is the reverse of what is commonly supposed. In the discovery of new facts, or the formation of new concepts, we commence with the conception of the concrete, and, the concept being formed, we find the name. But this, in the development of thought at which we have arrived, can occur only in the Physical Sciences. For, as we have observed, it is hardly probable that in the Moral Sciences any rudimentary thought can ever occur that has not already occurred to some one and been expressed in language. Hence, with regard to all matters dealt with in the Moral Sciences (as also in the Physical Sciences with regard to results already achieved), the order of our cognitions is, first, to learn the words,—i. e., the word-signs or words, words are the signs of notions. If, therefore, the notions (which form the basis of the whole) be confused and carelessly abstracted from things, there is no solidity in the superstructure. Our only hope then is in genuine induction" (Nov. Org., bk. i., aph. xiv).

The subject is more fully developed in aph. lix., and beautifully illustrated in aph. xcv. See also his doctrine of Idols, aph. xxxviii. et seq. It may be observed here, in passing, that no student of Philosophy, and still less of Logic, can afford to neglect the first book of the Novum Organum or the De Augmentis.
vocables,—and afterwards, the concepts or notions expressed in them.\(^1\)

§ 8. Our Supposed Knowledge often nonsense.—And as the latter function—outside the Exact Sciences—is in general very lamely performed, the result is that the greater portion of our supposed knowledge in abstract matters consists of words without definite notions attached to them, and is therefore merely nonsense. For when we reason with undefined or ill-defined terms we are dealing with mere delusions or dreams—like Ixion embracing clouds and begetting monsters. Thus, \(e.g.,\) when we assert, with Bentham and Austin, that General Utility is the ultimate test or principle by which the just and the unjust and right and wrong generally are to be determined, we are in fact talking nonsense; for it cannot be determined from this expression whether we have in view the welfare of a mere majority, or two thirds, or three fourths, or other proportion of mankind, and hence from this premise all sorts of extravagant opinions are deduced. Hence the mass of us

\(^1\)The logicians, from and including Hamilton, have entirely overlooked this distinction, and have thus substituted for the old logical doctrine of Simple Apprehension, the psychological doctrine of Conception,—a doctrine necessary to be understood, but which is concerned rather with the original formation of language than with its use as an instrument of reasoning.
generally, and all of us in many matters,—like Molière’s hero, who was surprised to find that he had been talking prose all his life,—have all our lives been talking nonsense.¹ And this is true not only of opinions commonly regarded as nonsensical, but of all opinions involving either undefined notions or notions to which there are no corresponding realities.

§ 9. The Critical Spirit Essential to Wisdom.—Our wisdom is therefore to be measured, not by the extent of our learning, or by knowledge of detached facts, or by vivacity of thought or expression, or by the confidence of our belief, but chiefly by the capacity to judge our supposed knowledge, and to detect its falsity or non-significance. In this way Socrates modestly explained the oracle of the Delphic god, that he was “the wisest of mankind.” For, he said, he alone had discovered that all men were ignorant, including himself; but others mistook their ignorance for knowledge.² We conclude, therefore, as we began, that what we regard as our knowledge consists mainly of unverified opinions or beliefs, and that however firmly these may be established,

¹ See Appendix C.

² As explained by Grote (cited infra, § 16, App. II), the thesis of Socrates was that “the natural state of the human mind” is “not simply ignorance, but ignorance mistaking itself for knowledge.”
or however passionately they may be asserted and believed, they do not necessarily, or even generally, constitute true knowledge. Hence, until we are enabled to distinguish the true from the false, we can have no assurance of their rectitude or truth.

§ 10. Logic the Ultimate Test or Criterion of Truth. — We must, therefore, seek some tests or criterions — if any there be — by which the truth or falsity of our beliefs may be determined; and of such two only can be conceived; namely, Experience and Reasoning, or Logic. Of these the former is more or less efficiently used by men in general; and in concrete matters and in the ordinary familiar affairs of life, its operation is moderately satisfactory. For thus, by actual contact with the hard facts of our experience, our opinions or beliefs are, to a large extent effectually, and often painfully, modified and corrected. But the function of experience is simply to furnish Reason with materials on which to work; and of Reasoning, or Logic, as Hobbes says: "So far are the mass of men from using it, that they do not even know what it is."  

"The most part of men, though they have the use of reasoning a little way, as in numbering to some degree, yet it serves them to little use in common life; in which they govern themselves, some better, some worse, according to their differences of experience, quickness of memory, and inclination to several ends; but especially according to good or evil
§ 11. The Decay of the Age in Logic and the Moral Sciences.—And this is true not only of the common people, but of the educated, and even of the philosophers and the professors; who in the last century, owing to the disuse of Logic, have in fact lost the very idea of it; so that in our schools and universities, under the name of Logic, anything but Logic itself is taught, and it has thus become a lost art.¹ Yet, obviously, in all abstract matters, and especially in Morality, Politics, and all the different branches of the Science of Human Nature, experience, while useful to us, can go but a little way, and therefore Logic must be an indispensable instrument. Hence it is to the disuse of Logic that the existing incoherent and chaotic state of the Moral Sciences is to be attributed.² It may therefore be confidently hoped that by the renewed use of Logic a revival of these sciences is to be anticipated, vying in extent with that of the concrete sciences in modern times, and fortune, and the errors of one another. For as for ‘science,’ or certain rules of their actions, they are so far from it that they know not what it is” (Lev., chap. v.).

¹ “We live in an age,” says De Morgan, “in which formal logic has long been banished from education; entirely we may say from the education of the habits.” The proposition is even truer of the present day; for in De Morgan’s time there still survived some of the old style of logicians.

² See Appendix D.
far surpassing them in practical utility to the human race.¹

§ 12. Of Authority and Prejudice.—I would not, however, in thus explaining and commenting upon the general dominance of authority and prejudice over men, be understood as altogether condemning it. Under existing conditions, and perhaps under all conditions, the opinions of the masses of mankind, in Politics and other matters of common concern, must be determined mainly by custom and authority. Hence the distinction made by the old philosophers between their esoteric and exoteric doctrines; the latter consisting of those that could be taught to the masses, the former, of those that required the peculiar training of the philosopher to comprehend—a profound distinction that has been lost in modern times. But though it may not be possible, or perhaps even desirable, to make all men philosophers, yet it is possible to make the masses of them logical in the matters with which they are con-

¹ The argument of Demosthenes in the first Philippic may be readily applied to the proposition asserted in the text: “First I say, you must not despond, Athenians, under your present circumstances, wretched as they are; for that which is worst in them as regards the past is best for the future. What do I mean? That your affairs are amiss, men of Athens, because you do nothing that is needful; if, notwithstanding you performed your duties, it were the same, there would be no hope of amendment.”
versant 1; and for those who aspire to be leaders of opinion, Logic is essential. For these, if worthy of the function to which they aspire, cannot afford to be deficient in this respect; they must either be logicians, or false prophets, or blind leaders of the blind.

§ 13. PLAN OF THE WORK.—Though I regard the study of Logic as essential to the cultivation and the use of the reasoning powers,—and hence as indispensable to the Moral Sciences,—yet it is chiefly as a test or criterion of fallacy that I propose to treat it. This use of it will, of course, necessitate some consideration of the elementary principles and rules of Logic as necessary to the understanding of the Doctrine of the Fallacies. But this part of my essay will be abbreviated to the utmost extent consistent with this object; that is to say, I will try to include everything essential to the understanding of the rudiments of Logic, but nothing more. If I should fail in this, and anything necessary should be omitted, the defect may be readily obviated by reference to the work of Whately, who, among elementary writers, may be regarded (in any true sense of the word) as the last of the logicians.

The subject will be treated in two books, the first entitled "The Analytic of Right Reasoning," the second, "Applied Logic"; the latter

1 See Appendix E.
of which will include two subjects, namely: "The Method of Logic" and "The Doctrine of the Fallacies," or "The Analytic of Wrong Reasoning." In treating of the last, the examples of the several fallacies will be taken almost exclusively from current theories of Politics and Morality. Our examples will therefore consist, not of mere trivialities, such as are so common in books on Logic, but of fallacies that, in perverting moral and political theory and in corrupting practice, have dominated, and still continue to dominate, the fortunes of mankind. They come to us, therefore, as veterans of what Hobbes calls the "Kingdom of Darkness," crowned with the laurels of victory.¹

¹ *Lev.*, chap. xliv.: see Appendix F.
BOOK I

THE ANALYTIC OF RIGHT REASONING
§ 14. Definition of Logic and of Involved Terms.—Logic is defined by Whately as the science and also the art of reasoning. Reasoning may be defined as consisting in the exercise of the comparative or discursive faculty of the mind—that is to say, the faculty by which our notions or concepts are compared with each other, and with the realities to which they are supposed to correspond, and their relations with each other, and with such realities are perceived. Or we may define reason as the faculty, and reasoning as consisting in its exercise. 1 But Logic—by which I mean the

1 The terms reason and reasoning, though conjugate, have unfortunately been divorced by logicians, and, following
traditional Logic—not to be regarded as having to deal with reasoning in general, but with explicit reasoning only, or ratiocination; which may be defined as reasoning expressed in language, or, so far expressed that the missing parts are understood. Hence it is rightly said by Whately that Logic is exclusively conversant with language; by which is meant, not merely the signs of thought, but also the thought signified. This follows from the definition, and also from considering the several subjects of which it treats, which, by the universal consensus of logicians, consist of the Doctrines of the Term, of the Proposition, and of the Syllogism. But all these are simply parts or kinds of language.

§ 15. Ratiocination Defined.—But Ratiocination, being a species of reasoning, must consist in the comparison of concepts or notions, and these, in order to fall within the province of Logic, must, ex vi termini, be expressed in terms. Hence, Ratiocination must be defined as consisting in the process of com-

them, by lexicographers generally; and accordingly Locke is blamed by Whately for confounding them. But in this Locke is right, and the logicians wrong; and the usage of the latter has been the source of infinite confusion in Logic. As I use the terms, Reason includes the faculties of Inference, Judgment, and Simple Apprehension; and Reasoning the corresponding processes.

1 See Appendix G.
paring terms, with the view of perceiving their relations. And this necessarily implies, also, the process of determining the meaning of the terms compared, or, in other words, the process of definition.

§ 16. Logic Defined.—Logic, regarded as a theory, may, therefore, be defined as the Analytic of Explicit Reasoning, or of Ratiocination—meaning, by this expression, the systematized results of an analysis of the processes involved in ratiocination.¹ And its practical end is to determine the meanings of terms and the relations between the concepts or notions denoted by them.²

§ 17. Of the Several Kinds of Terminal Relations.—The relations between terms are of two kinds, which may be called immediate and inferred; and the former, again, are of two kinds, that, for lack of better names, may be called intuitive and quasi-intuitive.

§ 18. The Intuitive Relations of Terms.—Of the former kind are all those relations between terms that are intuitively perceived upon comparing them together, as, e. g., the

¹ See Appendix II.

² "Knowledge [is] but the perception of the connection and agreement or disagreement or repugnancy of any of our ideas" (Locke, cited § 110 n. g. App. N). "Knowledge is not so much increased by a continued accession of new ideas as by perceiving the relations of those ideas which we have already acquired" (Eunomos, cited Chitty's Blackstone, introd. note).
relation of *species* and *genus* between the class of beings denoted by the term *man* and the class denoted by the term *rational*, or between the classes denoted by the terms *horse* and *animal*, or the relation of mutual exclusion existing between the terms of the proposition, "No two islands can be contiguous."

§ 19. **JUDGMENT DEFINED.**—The perception of a relation of signification between two terms is called Judgment; which may be defined as the intuitive perception of a significative relation between two terms. The result of the process is called a judgment; which may be defined simply, as a self-evident proposition.

§ 20. **THE QUASI-INTUITIVE RELATIONS OF TERMS.**—Analogous to the intuitive relations of terms are the relations between the terms of all assumed propositions, or assumptions; for these, though not intuitively true, are assumed or supposed to be such for the sake of the argument, and used as principles from which to reason; they may, therefore, be regarded as *quasi-intuitive*.¹ Under this head

¹ We borrow this form of expression from the lawyers, who find it indispensable, as, e. g., in the expressions *quasi-torts, quasi-contracts,* etc. As we are informed by Cicero, the Epicureans held that the gods had not bodies, but *quasi-bodies* only, *i. e.*, something like bodies. An Indian community, I have read somewhere, were much annoyed by a species of animal something like cows (*nielghais, I believe they called them*) that destroyed their crops, and the question
are included all the relations between the terms of propositions assumed as premises, whether upon authority, or from testimony, or otherwise, *i. e.*, between the terms of all propositions other than those that are intuitively perceived to be true, or that are inferred from other propositions.

§ 21. **The Inferred Relations of Terms.**—The inferred relations of terms include all relations that cannot be intuitively perceived from an immediate comparison of the terms, or that are not assumed, but that can be inferred by comparing the given terms respectively with a third or middle term, the relations of which to the given terms are known. Thus, *e. g.*, we may not be able to perceive from a mere comparison of the two terms, that "Logic is a branch of the Science of Language," but by comparing the two terms of the proposition respectively with the middle

arose whether it was lawful to kill them. The pundits to whom the question was referred were of the opinion that, though not cows, the animals were *quasi-cows*, and therefore not to be killed. The term will be found to be of equal utility in Logic as in the Law. In fact, a very useful book might be written on the subject—that might be appropriately termed *Quasics*. For, outside of concrete notions, all notions denoted by terms are formed by analogy from sensible images, and are *quasi-things* only, as, *e. g.*, *imagination*, *reflection*, *perception*, etc. We suggest the term *Quasics* not with a view of seriously recommending it for common use, but simply for the purpose of directing attention to a very important subject.
term, "The Science of the Term, the Proposition, and the Syllogism," the relation of species and genus between the subject and the predicate will be at once perceived. For "Logic is the Science of the Term, the Proposition, and the Syllogism," and "The Science of the Term, etc.," is a species or kind of "the Science of Language," and hence "Logic is a species or kind (i.e., a branch) of the Science of Language." And so we may not be able to perceive from a mere comparison of the terms that "the Thracians were barbarians," but by comparing these terms with the middle term, "Not-Greeks," the conclusion is apparent; for, ex vi termini, all "Not-Greeks" were barbarians. So, generally, using the letters X, Y, Z, etc., to represent the terms of any proposition, we may not be able to perceive intuitively the truth of the proposition that Z is X, yet, if it be intuitively perceived or assumed that Z is Y, and that Y is X, we may infer that Z is X.

§ 22. PROPOSITIONS AND SYLLOGISMS.—An immediate relation of terms, whether intuitive or assumed, can be expressed only in the form of a proposition—which may be defined simply as the expression of such a relation; and an inferred relation, only in the form of three propositions constituting what is called a syllogism. The proposition may be expressed in the formula: Y is X; and all syllogisms in the
formula: $Z$ is $Y$, $Y$ is $X$, $\therefore$ $Z$ is $X$; or, $Z$ is $Y$, $Y$ is not $X$ $\therefore$ $Z$ is not $X$—the letters standing for terms or names, and the three points ($\therefore$) being the sign of illation, and equivalent to the expression, "ergo," or "therefore." 1

§ 23. OF APODICTIC AND DIALECTIC.—Ratiocination may consist wholly of judgments and inferences, or partly of these and partly of assumed propositions. In the former case it is wholly illative, or demonstrative; in the latter,

1To define a term (as indicated in the etymology of definition) is in effect to establish the boundaries by which the class of significates denoted by it is separated from all other things; and these boundaries may be conveniently represented by circles or other enclosed figures. These are known as Euler's symbols, and are extremely convenient and universally used by logicians. A universal affirmative proposition is expressed by a circle contained in a circle, the former representing the subject, the latter the predicate; the universal negative by two circles excluding each other; and the syllogism, by thus expressing its several propositions; as, e. g., in the following diagrams:


\[\begin{align*}
&\text{Affirm. Syll.} \\
&\text{Neg. Syll.}
\end{align*}\]
only partially so, *i.e.*, only so far as the validity of the inference is concerned. The principles governing the former kind of ratiocination constitute what is called *Apodictic*; those governing the latter, *Dialectic*. It will be seen as we progress that Apodictic is far more extensive in its scope or use than is commonly supposed, and that it includes, in fact, not only the Mathematical Sciences, both pure and applied, but also a large part of Morality, Politics, and Jurisprudence generally. And especially, it is important to observe, it includes the subject of our present investigations. For Logic, though not so treated by modern logicians, is strictly a demonstrative science, and will be so treated in this essay.\(^1\)

\(\S\) 24. **Valid Ratiocination Illative in Nature.**—All ratiocination, or reasoning explicitly stated, discloses at once its validity or invalidity—that is to say, appears on its face to be either conclusive in its effect, or fallacious. Hence, all ratiocination, unless fallacious, is illative or conclusive, or, we may say, demonstrative in its nature. On the other

\(^{1}\) One of the most universal infirmities of the average mind is an incapacity to distinguish (outside the mathematics) between mere *opinion* and *apodictic*, or demonstrated *truth*. With regard to the latter, the man who is conscientious and accurate in his Logic may realize the fine saying of Seneca: "It is truly great to have in one the frailty of a man and the security of a god" (cited Bacon, *Essays*, "Of Adversity").
hand, unless explicitly stated, no reasoning, however apparently convincing, can be regarded as of this nature. Hence, from a logical point of view, reasoning in general may be regarded as either valid (i.e., illative), or as invalid; the latter of which may be either fallacious or simply inconsequent. The former may be appropriately called Logical Reasoning, the latter Non-logical or Rhetorical; by which is meant not necessarily illogical or fallacious, but either fallacious or simply inconsequent, i.e., non-illative.

§ 25. Right Reasoning Defined.—It is with the former only that Logic is directly concerned, and to it we may without impropriety give the name of Right Reasoning. For the logical quality of the reasoning does not depend upon the truth or falsity of the conclusion, but upon the rectitude of the definitions, judgments, and inferences.

§ 26. Logic the Art of Right Reasoning.—Logic, therefore, regarded as an art, may be simply defined as the Art of Right Reasoning; and it must therefore be regarded as denoting the ultimate test or criterion of truth or error. For until the reasoning is made explicit, it cannot be determined whether it is right or otherwise. It also includes the doctrine of Fallacy, or Wrong Reasoning; but as the latter has for its end simply the
avoidance of error, as a means of assuring the rectitude of our reasoning, it may be regarded simply as one of the practical aspects of the doctrine of Right Reasoning.

§ 27. LOGIC TO BE REGARDED AS INTELLECTUAL MORALITY.—Logic must, therefore, be regarded as bearing to reasoning the same relation as Morality to conduct. It may, therefore, be appropriately called *Intellectual Morality.*

1 Hence it is that Logic, like Morality, is not popular with those who disregard its precepts; among whom are to be included the large majority of writers, and especially of philosophers. The principle is as expressed in the adage:

"What thief e'er felt the halter draw
With good opinion of the Law?"
CHAPTER II

DOCTRINE OF THE TERM

I

OF THE NATURE OF THE TERM

§ 28. "Term," "Name," and "Word" Distinguished and Defined. — These words are often used as synonymous, but the distinction between them is material and important. A word is a vocal sign, or vocable, expressing a thought, or a thought expressed by such a sign. Under the name "word" is included the substantive or noun, and also other parts of speech, as, e. g., the article, the conjunction, etc. A name (noun or substantive, which may be either simple or complex) is a word or set of words used to signify an object of thought regarded as a thing, i. e., as an existing substance or entity. The knowledge or cognition of a thing by the mind is called a notion or concept; hence a name may be otherwise defined as a word, or set of words, expressing a notion, or
as a notion thus expressed. A notion or concept is itself a thought, but it differs from other thoughts as being the thought of a thing, i.e., of something as existing. A term is a name used as a subject or predicate of a proposition. It is therefore to be regarded merely as an element of the proposition; and the proposition as the principal subject in Logic.

§ 29. "THING" DEFINED.—The term thing is used in two different senses that must be carefully distinguished. In its proper sense the term denotes an actual thing or substance, whether material or spiritual, as, e.g., mineral, vegetable, animal, gas, man, soul, God, etc. In this sense things constitute the actual universe, and all notions or concepts whatever, unless false or unreal, are ultimately derived from them. But, in another sense, the term is used to denote, not only actual existences, or, as we may call them, real things, but mere objects of thought, or things existing only in contemplation of mind, and to which there are, in fact, no real things directly corresponding.¹ These may be appropriately

¹All true or real notions must correspond to real or actual things, but the correspondence may be either direct between the notion and the real things signified by the term— as in the case of concrete terms, e.g., "man," "horse," etc.; or indirect—as in the case of abstract terms— between the notion and the things whose attributes are signified. Thus, taking for example the term "redness," there is apparently a
THE TERM

called quasi-things; and of this kind are the concepts or notions denoted by all abstract terms; which denote, not real things or individuals, but mere abstractions, as, e. g., such terms as "justice," "the state," the names of the several colors, disease, death, etc.; where the things denoted are not actually existing things, but mere concepts of qualities or attributes of things objectified by the mind.

§ 30. "Concept," "Notion," and "Thought" Defined.—The term "concept," or "notion," or "thought" (in this connection we may use either indifferently) is a relative term implying or connoting, in its strict or proper sense, an individual thinking mind of which it is the product; and hence the term will have a different meaning according to the correlative to which it refers. It must therefore have many different senses; of which two must be especially distinguished. In its proper sense it denotes simply a certain affection of the mind of the individual; and in this sense, obviously, it is momentary and evanescent,—like the snow falling on the river, described by the poet, as "ae moment white,

direct correspondence between the notion expressed and the quasi-thing signified, though in reality they are the same; but there is an indirect real correspondence between the notion of redness and the red things of which it is a quality.
then gone forever.” For though, it is said, the thought recurs to us, it is not, nor can it be, the same thought, but is merely a copy or image of it. So, when a thought—as it is said—recurs to us, it is always, or at least almost always, suggested to us by the word in which it is embodied; and, as to us, so also to others. But Logic does not have to deal with the momentary, fleeting thought of the individual, but with the thought only that is continuously, or we may say permanently reproduced, and communicated by one to another; that has become incarnate in words, and is thus, even when lost from the mind, at once preserved, and continuously suggested, or brought back to the consciousness of each and all. Hence, in Logic, the terms, notion, concept, and thought, are to be regarded as used in a secondary or derived sense, as denoting the common notions, concepts, and thoughts of mankind embodied in words. Hence the things or signifcates denoted by abstract and other universal terms have in fact a kind of existence outside of any and all individual minds; which, as opposed to substantial, may be called logical existence; i. e., they exist in the word (logos), and their existence is as real and of precisely the same nature as that of the word of which they are an essential part. Hence, though we speak of abstractions as fictitious
(i. e., feigned) or imaginary things, yet they are real, and in some cases, as, e. g., in the case of death, disease, misery, poverty, etc., terribly real facts. What is meant by the term "fictitious thing" is, not that the notion signified is false or unreal, but that, for logical purposes, it is fictitiously regarded as a thing.

§ 31. The Normal Logical Term.—Every term legitimate for logical purposes, or we may say every logical term, is therefore to be regarded as involving or implying three essential notions or elements, namely: (1) the vocal sign, or vocable, (2) the notion denoted, and (3) the actual things, or objective realities, to which the notion and the vocal sign are supposed to correspond. These are all to be regarded as, in one sense, essential elements of the logical term. For though, where the last is lacking, a term may exist, and it is, therefore, possible to have an absurd or nonsensical term, yet such a term is not such as is contemplated when we regard the end of Logic; which is not to deal with absurdities or ingenious puzzles, but to discover truth and avoid error. Hence, an absurd or nonsensical term, or, in other words, a term whose signification does not correspond to reality, is not the normal or true term, essential to legitimate ratiocination; nor is Logic—unless in illustrating some of its formal operations—in any way concerned with
it, except to detect and expose its inherent vice and its essential insufficiency for logical purposes.

§ 32. The Denotation and Connotation of Terms.—All terms are regarded in Logic as denoting or signifying classes of individuals.¹ The individuals constituting the class denoted by the term are marked or distinguished by certain common attributes, at once common and peculiar to the class, as, e.g., the class "man" by the mark "rational," by which it is distinguished from other kinds of animals. Accordingly a term is said to denote the individuals designated by it, and to connote the qualities or marks by which the class is determined. Thus, e.g., the term "man" denotes the class of animals known by that name, and connotes the quality or attribute of rationality by which the class is distinguished.

§ 33. The Meaning and Signification of Terms.—The individuals constituting the class denoted by a term are said to be signified by the term, and are called its significates. Thus the term, man, denotes the class, man, as a whole, but signifies each and all of the individual men composing it. The significates of a term may be real,—which is the case when they are real individuals or things, existing in

¹ See infra, § 35.
nature; or they may be unreal, or fictitious, i. e., existing only in contemplation of mind; which is the case with all abstract terms, and with concrete terms where the classes of individuals denoted are fictitiously regarded as individuals,—as, e. g., when we speak of "man" as one of the significates of "animal." When a term denotes a class of real individuals — as, e. g., "man," regarded as denoting men generally—its significates are real; when it denotes a class of lower classes—as, e. g., the several races, Asiatics, Europeans, etc.—they are unreal or fictitious. In the former case the term is said to denote an insima species; which is to be defined as a class made up of real individuals. By the meaning of a term is meant both its denotation, or signification, and its connotation taken together; and the word "meaning" may also be regarded as equivalent to notion or concept.

§ 34. The Extension and Intension of Terms.—The extension of a term corresponds to its denotation, or signification, and is determined by the extent of the class denoted, or by the number of significates signified by it. The intension of a term is but another name for its connotation,—both words denoting merely the qualities or attributes, or, in other words, the marks by which the class is determined.
II

OF THE SEVERAL KINDS OF TERMS

§ 35. SINGULAR, AND COMMON, OR UNIVERSAL, TERMS.—Grammatically speaking, terms are said to be either singular or common, or, as otherwise expressed, singular or universal. A singular term is one that denotes an individual or single thing, as, e. g., any particular thing, animal, or man. A common or universal term is one that denotes either a class of individuals or a class made up of other classes. But in the latter case, the subordinate classes may be regarded as individuals constituting the superior class; and conversely the individual may always be regarded as a class,—i. e., a class of one. 1 In this work, therefore, the distinction between singular and common or universal terms will be regarded as logically immaterial; all terms will be regarded as universals, or, in other words, as denoting classes of significates.

§ 36. ADJECTIVES.—Hence also adjectives used as terms will be regarded as nouns or sub-

1 "By a class is usually meant a collection of individuals . . . ; but in this work the meaning of the term will be extended so as to include the case where but a single individual exists, as well as cases denoted by the terms 'nothing' and 'universe': which as 'classes' should be understood to comprise respectively 'no beings' and 'all beings.'" — Boole, Laws of Thought, p. 25.
stantives; that is to say, where a term is in adjective form (which can occur only with the predicate) it is either regarded as a substantive, or converted into one by adding the substantive understood. Thus, e. g., the proposition, "Man is mortal," is to be read: "Man is a mortal," or "a mortal being." ¹

§ 37. Abstract and Concrete Names.—A concrete name is one that denotes a class of real individuals. An abstract name is one that denotes qualities or attributes conceived as existing apart from the things in which they inhere, or, in other words, fictitiously regarded as things,—as, e. g., whiteness, strength, goodness, humanity, etc.² Abstract names are commonly singular in form, but in their essential nature they are always universal. Thus, when we speak of virtue, the name is to be regarded as

¹ "If we attach to the adjective the universally understood subject, 'being' or 'thing,' it becomes virtually a substantive, and may for all the essential purposes of reasoning be replaced by the substantive. Whether or not in every particular of the mental regard it is the same thing to say, 'water is a fluid thing,' as to say, 'water is fluid,' it is at least equivalent in the expression of the processes of reasoning."—Boole, Laws of Thought, p. 27.

² The distinction between concrete and abstract names corresponds precisely to the distinction made by old logicians between names of first intention and names of second intention. The former are names that denote real significates; the latter, names that denote fictitious significates, or quasi-things. See further on this point Appendix I.
denoting, not a quality existing in any particular man, or in itself, but the class of qualities by which all virtuous men are distinguished. So, though we may consider the color red, or redness, in the abstract,—dismissing from the mind the individuals in which it is manifested,—yet, upon analyzing the concept, we cannot fail to perceive that there are as many individual instances of red, or, we may say, as many individual reds or rednesses, as there are individual things in which the color is manifested; and that red, or redness, is simply the denomination of the class of colors thus manifested. Hence, abstract names, though grammatically singular, are to be regarded as plural, and as differing from concrete names only in this, that the individuals constituting the class are qualities,—i. e., quasi-things, or fictitious, not actual existences,—and that among the marks by which the class is distinguished are the actual individuals in whom alone the qualities exist. An abstract name is therefore to be regarded as denoting a class of qualities; and as connoting the individuals in which they inhere.

§ 38. The Distinction of Fundamental Importance.—The distinction between concrete and abstract names, or names of first, and of second intention, is one of fundamental importance. In dealing with the former, the things denoted by the names we use are ever
present to the mind, and we may therefore, as is asserted by Mill, be said — without violent absurdity — to deal with things, rather than with notions or names. But where we deal with abstract terms, the things present to the mind are mere abstractions, fictitiously regarded as things; and we are, in fact, dealing not with things, but with quasi-things only.  

§ 39. Positive and Negative Terms.—The distinction between positive and negative terms is also one of fundamental importance in Logic. By this division of terms the whole universe of things, real and fictitious, is divided into two classes, the one marked by having, the other by not having, a certain quality or qualities, as e.g., white things, and things that are not white; and it is obvious that to each positive there must be a corresponding negative term.

§ 40. Of the Universe of the Proposition.—But ordinarily in speech we have in view a more limited class, and must be understood to refer, not to the universe of things, but to some class less than the universe, but superior to the classes denoted by the subject and predicate; and this superior class is said to constitute the universe of the proposition in which the terms are used. Thus, when we speak of "mortal" and "immortal," the class

1 See Appendix K.
of "living things" or "beings" is obviously referred to as the superior class, and is, therefore, said to constitute the universe of the proposition; and the division is to be understood to be into "mortal" and "immortal" beings. So, in the proposition, "Brutes are irrational," the superior class we have in view is that of animals, and this class is to be regarded as the universe of the proposition; as (denoting "not" by the Greek privative, $a$) may be illustrated by the following diagrams, either of which may be used:

III

OF THE ANALYSIS OF TERMS

§ 41. Apprehension.—As it is the function of Logic to compare the notions denoted by terms, with the view of determining their relations, a preliminary process is essential. namely, that of apprehending or understanding the significations of terms; which is called by logicians, "Simple Apprehension." 

1 The operations of the mind involved in reasoning are (1) Simple Apprehension, (2) Judgment, and (3) Inference (see
This is effected by means of what may be called the "Analytical Processes"; which will next be considered.

§ 42. Analytical Processes. — As preliminary to apprehension, it is essential that the sense in which the term is to be used shall be identified, or, in other words, that of the several senses usually denoted by a vocable, one shall be selected. This is often called nominal definition (i.e., definition of the name), but improperly; for until it is determined in what sense a term is used, there is in fact no name. Hence we call it, Vocal Definition, i.e., Definition of the Vocable. Next, it is necessary, before the two terms can be compared, to apprehend, in the case of each of them, the significates of the term, or the class of significates denoted by it; for otherwise we will not be able to compare their significations. This is effected by the definition of the term; which, to distinguish it from vocal, is called nominal or real definition¹; and this again involves the process of classification or division.

Whately, Logic). I have altered the ordinary statement of these operations by substituting for the third "Inference" instead of "Discourse"; which is commonly defined as "reasoning" or "ratiocination." But, as used in this work, these words include both Apprehension and Judgment.

¹ There is some confusion among logicians as to the use of the terms, Nominal Definition and Real Definition. By some, the former term is used as denoting what I have called vocal
§ 43. **Vocal Definition.**—A word, or vocable, *i. e.*, the vocal sign, has usually many significations; and commonly, in using it, we do not, at first, distinguish between such of the notions denoted by it as are nearly the same, but, instead of regarding it (as we should) as part of several names, use it as though it were a single name. But in thus using a vocable without distinguishing its several senses, it is inevitable that, in the course of the ratiocination, it will be used in a shifting sense, or rather, we should say, in several senses, as suggested by the varying occasion; and that the coherency of our reasoning will thus be destroyed. This fault in ratiocination is called the fallacy of *confusion* or of *ambiguity*, and, as will be seen in the sequel, is one of the most common and most serious of fallacies. Hence it is one of the most important and imperative of logical rules that, in the case of every word we have occasion to use in our reasoning, the sense in which it is to be used shall be clearly defined; but this seems to be incorrect. According to the better usage, a Nominal Definition is a definition of the Notion expressed in a term; and hence Whately says "that Logic is concerned with *nominal* definitions only." To this Mansel objects on the ground that "Logic is concerned with *real* or notional definitions only; its object being to produce *distinctness* in *concepts*, which are the *things* of Logic" (Mansel's *Aldrich*, p. 39). But this is precisely what Whately means; and says.
distinguished and consistently observed. And this indeed, *ex vi termini*, is essential even to the beginning of ratiocination; for, until this is effected, we have not even that essential material of ratiocination, a *name*, with which to deal. The vocal definition of a term may be effected in various ways,—as, *e. g.*, by the use of any other term, or phrase, or sentence of equivalent signification; or, negatively, by rejecting those senses of the word that we do not wish to use; or, often, by an imperfect definition, as by simply specifying the genus of the class denoted by the term; or, in fine, by any means that may serve to confine the term to one sense only, and thus to prevent ambiguity.

§ 44. **Division and Classification.**—Division consists in distributing the class of significates denoted by a name into subordinate classes, with appropriate names; classification in the reverse process of assigning a class denoted by a name to a class denoted by another name.

§ 45. **Genus and Species.**—In the former case, the class distributed is called the *genus*; the classes into which it is distributed, *species*. In the latter, the class assigned is a *species*, the class to which it is assigned, the *genus*. The genus and species, however, as in the case of synonyms, may be of equal extension.
§ 46. DIVISION.—Division is an act of Analysis; Classification, of Synthesis. But the same principles govern both, and the elucidation of one will equally explain the other. In Logic, the analysis of terms is the more important process, and we will therefore adopt, as the subject of explanation, the process of Division. The term to be divided, or, rather, the class denoted by the term, is, as we have said, called the genus; the subordinate classes into which the genus is divided, species. The species must, of course, be exclusive of each other,—i.e., they must not overlap; and taken together they must exhaust the genus. Thus, the term thing—meaning thereby things and quasi-things—may be divided, and subordinate classes subdivided, as follows:

\[
\begin{align*}
\text{Things} & \\
\text{Real Things} & \quad \text{Quasi-Things} \\
\text{Bodies} & \quad \text{Not Bodies} \\
\text{Organic} & \quad \text{Inorganic} \\
\text{Animal} & \quad \text{Not Animal} \\
\text{Rational} & \quad \text{Not Rational} \\
\end{align*}
\]

§ 47. DICHTOTOMY.—It will be observed that the above division is, in each case, two-
fold,—*i.e.*, into two classes, represented by a term and its negative. This is called *Dichotomy*, and, as in using it we are less liable to error than in other modes of division, it is most commonly used. The genus may, however, be divided into three or more species, provided the species taken together exhaust the genus, and be exclusive of each other,—as, *e.g.*, in the division of Bodies into (1) Inorganic, (2) Vegetable, and (3) Animal.

§ 48. **Nominal Definition of Terms.**—The definition (*i.e.*, the *real* or *nominal* definition) of a term consists in assigning the class denoted by it to an appropriate genus, and giving its *specific difference*; by which is meant some mark or marks peculiar to it, by which it may be distinguished from other species. It is, therefore, a species of classification,—*i.e.*, it consists simply in classifying the given class, or species, by assigning it to a genus, and in adding also the appropriate marks, or *specific difference*, by which it is distinguished from the other species contained in the genus. The definition of a term is, therefore, to be regarded simply as a complete classification of it; and the classification of it as an incomplete or imperfect definition. But the latter has the advantage that it can often be used where the former would be inconvenient or impossible.

§ 49. **The Essence of the Term.**—A
quality at once common and peculiar to the individuals denoted by a term is called a property of the class denoted; a quality common to the class, but not peculiar to it, is called an accident. The definition of a term is made up by selecting from the accidents of the term one to serve as a mark for the purpose of determining the genus, and from the properties one to serve as specific difference. These together constitute the essence of the term; which will therefore vary with the definition, and be determined by it. Thus, e. g., if we define man as a rational animal, "animal" will be the genus; "rational" the specific difference; "talking," "laughing," "cooking," etc., properties; "mortal," "carnivorous," "mammal," etc., accidents. But we may, if we choose, define him variously as a talking, laughing, or cooking, mortal, carnivore, or mammal. The essence of a term is therefore but another name for the meaning of the term. Properties not used for specific difference, and accidents not used for genus, do not enter into the essence of the term.

There is much confusion among logicians in the use of the term accident. The definition in the text is that of the best authorities, including Aristotle; and the term should be consistently thus used.
CHAPTER III

DOCTRINE OF THE PROPOSITION

I

RUDIMENTS OF THE DOCTRINE

§ 50. Proposition Defined.—A proposition may be defined as the expression of a relation of signification between two terms; which, of course, implies the expression of the corresponding relation between the notions expressed in the terms.

§ 51. The Grammatical Proposition.—But here there is a difference between Logic and Grammar, or, we may say, between the logical and the grammatical proposition. In the latter, any of the innumerable relations existing between terms, or, what is the same thing, between the things denoted by them, whether past, present, or future, may be expressed as existing between the terms; and the relation may be expressed by any copula or connecting word, or the same word may be
used to express both copula and predicate, as, *e.g.*, "John struck William"; "The sun will rise at six o'clock to-morrow"; "It rains"; "The Carthaginians did not conquer Rome," etc. But in Logic the only copula used is the present tense of the verb "*to be,*" with or without the negative particle; and the only interterminal relation considered is that of species and genus; which may be either affirmed or denied.

§ 52. **The Logical Proposition.**—Accordingly the logical proposition is of two forms, the affirmative and the negative. In the former the relation of species and genus between the terms is affirmed,—as, *e.g.*, "Man is mortal," "Y is X," etc.; in the latter it is denied,—as, *e.g.*, "Man is not perfect," "Y is not X," etc. The affirmative proposition may be read, either, "Y is X," or "Every Y is X," or "All Y's are X's"; or, to take the concrete example, "Man is mortal," or "Every man is mortal," or "All men are mortal,"—these expressions being all equivalent, and signifying equally that the subject class—or class denoted by the subject—is a species of the predicate class. The negative proposition may be read either as above or as follows: "No man is perfect," "No Y is X," etc. It is a cardinal postulate in Logic that all propositions may, and indeed—for purposes
of logical analysis — must be converted into logical form; as, e. g., the above examples into the following: "John is the man who struck William"; "Six o’clock is the hour at which the sun will rise to-morrow"; "Rain is falling"; "The Carthaginians are not [or, grammatically, we should say, "were not"] the conquerors of Rome."

§ 53. INTERPRETATION OF THE LOGICAL PROPOSITION.—In all logical propositions the copula is to be interpreted as meaning "is contained in" or "is a species of," or the contrary, as the case may be. 1 Hence in Logic the only

1 There are commonly recognized by logicians four forms of the proposition, designated respectively by the letters, A, E, I, and O, and called the "Universal Affirmative," the "Universal Negative," the "Particular Affirmative," and the "Particular Negative" (see infra, § 88). But if in I and O we regard the expression "some Y" — instead of "Y" — as the subject of the proposition, these forms will become the same as A and E. Hence, propositions may, as in the text, be regarded as of two kinds only, namely, affirmative and negative; the former affirming that the subject is included in the predicate class; the latter denying that it is so included. This distinction agrees precisely with our definition, and will be sufficient for our present purposes, and, indeed, for all practical purposes.

2 The affirmative proposition "Y is X" is to be construed as asserting that the class Y is wholly included in the class X; the negative, "Y is not X," that it is wholly excluded. But the class Y may denote a part of a class, as, e. g., "Some A"; in which case the proposition "Y is X," or "Y is not X," would be equivalent to the ordinary forms, "Some A is X," or "Some A is not X,"
significative relation recognized is the relation of the inclusion or exclusion of the subject class in or from the predicate: and accordingly this may be called appropriately the *logical relation*. Yet the logical proposition is not less capacious of expression than the grammatical; for, as the latter may always be converted into the former, it follows that all relations may be expressed in the one as in the other. The only difference is that in the grammatical proposition the relations between the notions involved may be expressed either in the copula, or in the terms themselves; while in the logical proposition the only interterminal relation expressed (*i. e.*, affirmed or denied) by the copula is that of species and genus, and all other relations between notions are expressed in the terms,—*i. e.*, in complex terms.

§ 54. The Conversion of Propositions. —By *conversion* is meant the transposition of subject and predicate—*i. e.*, making the predicate the subject, and the subject, predicate. But, such conversion, to be legitimate, must be illative, *i. e.*, the force or conclusiveness of the proposition must not be affected. Thus the proposition, ""Y is not X"" (since the subject and predicate classes are mutually exclusive), may be converted into the proposition, "" X is not

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1 This is admirably illustrated by Mr. Boole's system of signs, of which I append an epitome. See Appendix L.
which is called simple conversion; and so with all definitions, and other equational propositions; and also with the particular affirmative proposition, "Some Y is X." But the affirmative proposition, "Y is X," cannot be thus simply converted; for the subject class is identical with only "some" of the predicate class, and in conversion the predicate must be qualified by that particle, thus substituting a new term. Or, symbolically, the proposition, "Y is X," can be converted only into the proposition, "Some X is Y"; which is called conversion per accidens.

II

SEVERAL THEORIES OF PREDICATION

§ 55. The Copula.—In the logical proposition, as we have seen, the copula is interpreted as meaning "is contained in," or the contrary; and this is the traditional, or, as it may be called, orthodox, theory of predication. But the copula may be otherwise interpreted; and from these several interpretations several theories of predication will result. Of these, two may be distinguished as requiring some remark, namely, the Equational Theory, in which the copula is interpreted as meaning, "is equivalent to," and is expressed by the sign of equivalence (=); and the Intensive Theory, where it
is interpreted as meaning, "has the quality or attribute." Thus, e. g., the proposition, "Man is rational," is interpreted according to the *Traditional Theory* as meaning, "the class man is contained in the class rational"; according to the *Equational Theory*, as meaning, "the class man is the same as the class rational"; and according to the *Intensive*, as meaning, "the individuals constituting the class man have the quality or attribute, rational, or of rationality."

§ 56. **The Equational Theory.**—In the logical proposition, the classes denoted by the subject and predicate may be equal; for, where this is the case, each may be said to be contained in the other. Hence in such cases the proposition is always convertible, as, e. g., we may say indifferently that "man is a rational animal," or that "a rational animal is a man," or, generally, if \( Y = X \), either that "Y is X" or "X is Y." Such propositions are recognized and used in the traditional Logic, as in the case of definitions, and in other cases, but it is not thought necessary to express the equivalence of the terms. Hence in the affirmative proposition "Y is X" it cannot be determined from the form of the proposition whether X is of greater extension than Y, or of the same extension.

§ 57. **Quantification of the Predicate.**
—The modern doctrine of "the quantification of the predicate" has for its object to remedy this supposed defect by expressing in every proposition by an appropriate sign the quantity of the predicate, or, in other words, by indicating whether it is distributed or not; and this is effected by prefixing to the predicate a sign indicating the relation of quantity between it and the subject, and giving to the proposition an equational form. Thus, e. g., the proposition, "Y is X," may be expressed in the form "Y = vX," which is the method of Boole; or in the form "Y = YX," which is the form proposed by Jevons, and is read, "Y = the part of X that is Y," or "the Y's are the X's that are also Y's." Or, more simply, instead of the proposition, "Y is X," we may say, "Y is a certain species of X"; or, to take a concrete example, instead of the proposition, "Man is an animal," we may say, "Man is a certain species or kind of animal." Hence, whether an equational proposition shall be expressed in the traditional or in the equational form is a matter of choice to be determined by convenience. Generally the

1 A term is said to be "distributed" when it is taken universally, i. e., where the other term of the proposition is, or may be, predicated of all the individuals denoted by it, as, e. g., the subject of a universal affirmative, or either subject or predicate of a universal negative proposition (see § 87).
traditional form is sufficient, as we can readily determine from the matter of the proposition whether it is to be regarded as equational or otherwise. But in the mathematics the equational form is much the more efficient, and is therefore always used.

§ 58. The Intensive Theory.—The difference between the traditional and the intensive theory of predication is that, in construing the proposition, we have regard in the former to the extension of the terms only; but in the latter, in construing the predicate, we have regard to its intension. Thus, when we say "Man is mortal," we mean, in the former case, that the class man is contained in the class mortal; but in the latter, that man has the quality or attribute of mortality. But the latter expression means nothing more than that "the quality of mortality is contained in, or among, the qualities of man"; which is itself an extensive proposition. Hence the intensive interpretation of the proposition simply results in an extensive proposition in which the qualities of the original terms are substituted for its original significates, and the terms inverted. Thus, e.g., if we denote by Y' the qualities of Y, and by X' the qualities of X, the proposition, Y is X, may be converted into X' is Y'; which may be called Intensive Conversion, or conversion by Intensive Interpretation.
§ 59. Traditional Theory of Predication.—Even under this theory the proposition seems to be susceptible of several interpretations. Thus, e. g., we have interpreted the copula as meaning "is contained in" or "is a species of"; and again we may interpret it as meaning that the significates constituting the subject class may each and all be called by the name constituting the predicate—or, in other words, that the name predicated belongs to the significates of the subject term, or of any of them; which has been called interpreting the judgment "in its denomination" (Thompson’s Laws of Thought, § 195). But for all logical purposes these interpretations are practically the same, and it will make no difference whether the proposition be interpreted in the one way or the other. This is sufficiently obvious with regard to the expressions, "is contained in," and "is a species of"; and is equally true of the interpretation suggested by Dr. Thompson. For, taking as an example the proposition, "Man is an animal," it is obviously indifferent whether we construe it as meaning "the class man is included in the class animal," or that "it is a species of the class animal," or that "the name animal is applicable to all significates of the name man." These varieties of interpretation will, therefore, not demand a further consideration.
§ 60. Collective and Distributive Interpretation.—There is, however, another difference of interpretation it is important to consider; and especially with reference to mathematical reasoning, which is to be considered presently. Common terms, or terms denoting classes of more than one, may be used either collectively or distributively,—i.e., the class denoted by the term may be regarded either as a whole made up of individuals,¹ or as a number of individuals constituting a class, or signified by the name. Thus, e.g., the term "man" may be used to denote either the class "man," as when we say, "Man is mortal"; or the individuals composing the class, as when we say, "A man is a mortal," or "Men are mortals." Whether a term is used collectively or distributively may be indicated, as in the above examples, by the expression, or may be simply understood; or the expression may be such as not to indicate either expressly or implicitly whether the term is used in the one way or the other. With regard to the subject of the proposition it is logically immaterial in which way the term is used. Thus, in the proposition, "Y is X," the subject is used collectively; and in the proposition, "All Y's are X's," or

¹ When a concrete term is construed collectively, it becomes abstract, and is to be regarded as denoting, not a number of real individuals, but one quasi individual only.
“Every Y is an X,” or “A Y is an X,” distributively; but the forms are logically equivalent.

So with regard to the predicate, where the terms are of equal extension, it is immaterial whether it be construed collectively or distributively, provided, if the predicate be construed collectively, that the subject also be thus construed. For to construe a term collectively is to regard the class denoted by it as an individual, and a term thus construed is therefore to be regarded as a singular term. But a singular term cannot be predicated of any but a singular term, with which it must exactly conform in signification; or, in other words, a singular term can be predicated of another singular term only in the equational proposition. Thus, e.g., in the proposition, “Y is X,” it is immaterial whether we regard Y as denoting the class Y, or as signifying the significates composing the class. But the class X cannot be construed collectively unless we also construe the class Y in the same way, and unless also the two classes are co-extensive, or, in other words, unless the proposition can be put in the form, Y = X.

III

OF THE PREDICABLES

§ 61. DEFINITION AND DIVISION OF THE PREDICABLES.—A predicable may be defined
as a term that may be made the predicate of an affirmative proposition. As explained above, such propositions may be either *equational* or *non-equational*. In the former case the predicate is of the same extension as the subject; in the latter, of greater extension. All predicables, therefore, may be divided into two classes,—namely, those that are equivalent to the subject, and those that are not equivalent. An equivalent predicatable may be either *definition* or *property*; for each of these is precisely co-extensive with the subject (§ 49). Non-equivalent predicables must be either *genera* or *accidents*; either of which may always be predicated of the subject (*Ib.*). This is the division of predicables used by Aristotle.

§ 62. Twofold Division of Predicables.—But the distinction between "definition" and "property" seems, with relation to the subject of predicables, to be unimportant; for "**property**" differs from "**definition**" only in the use made of the former (*Ib.*). And so with reference to the distinction between *genus* and *accident* (*Ib.*). Hence it has been proposed "to abandon, as at least unnecessary for logical purposes" (or rather, we should say, for purposes of predication), "the distinctions between *property* and *definition*, *genus* and *accident*, and to form, as Aristotle has also done, two classes of predicables; one of predicables taken
distributively and capable of becoming subjects in their respective judgments without limitation; the other of such as have a different extension. In the former the predicatable has the same objects [i.e., significates] as the subjects, but different marks, or a different way of representing the marks. In the latter there is a difference, both in the marks and the objects” (Thompson’s *Laws of Thought*, § 69.)

§ 63. **ONE KIND OF PREDICABLES ONLY.**—But even the twofold division of predicables, into *equivalent* and *non-equivalent*, is, from the traditional standpoint, of minor importance; for, as we have seen, the old Logic ordinarily takes no account of equational propositions, but these, like others, are regarded as importing simply the inclusion of the subject in the predicate; and in this mode of interpreting the proposition, we have, in effect, a complete doctrine of the predicables.

1 The division of predicables most commonly used is that of Porphyry (Aristotle’s *Logical Treatises*, Bohn’s edition, Introduction of Porphyry; also Jevons’s *Lessons in Logic*, p. 98). According to this division, “*Specific Difference*” is substituted for the “*Definition*” of Aristotle’s division, and there is added as a fifth predicatable, “*Species,*” as being predicatable of individuals. But, as observed by Mansel (Aldrich’s *Logic*, Preface), “whether this classification is an improvement, or is consistent with the Aristotelian doctrine, admits of considerable question.” The view taken in the text is in every respect preferable (Thompson’s *Laws of Thought*, pp. 136 et seq.).
§ 64. Of the Relations of Terms Generally.—The end of Logic is to determine the relations, and, as involved in this, the definitions, of terms, or (what is the same thing), of the notions expressed in terms (§ 16). Of these notions, the most conspicuous are those existing between what are called relative words—as, e. g., father and son, wife and husband, higher and lower, etc., and also the active and passive forms of the verb, and all inflections of verb or noun, or, in a word, all paronyms, etc. But the term, relative, though applicable, is not peculiar to this class of words, and is, therefore, not altogether appropriate. Relations, more or less apparent, exist between all terms, and in the development of these consists the raison d'être of Logic. Hence, properly speaking, no term can be said to be absolute, as opposed to relative. For—to consider only one of the most general of relations—any thing, or class of things (real or fictitious), must always be assignable to one of two classes, namely the class denoted by a given term, or to the class denoted by its negative; and, in addition to this universal

1 This, of course, is true only on the assumption that we reject Particular Propositions, as proposed (§ 52, note).
relation, there are numerous others, either of a
general character,—as e. g., the relation be-
tween numbers, or other expressions of quan-
tity,—or such as are peculiar to certain words,
—as, e. g., between hunger and animal, hunger
and edible, gravity and body, fish and water,
the sun and the planets, etc. In fine, the re-
lations between terms are innumerable, and,
when the significations of terms are appre-
hended, these relations may, in general, or, at
least, in innumerable cases, be either intu-
tively perceived, or demonstratively inferred.

§ 65. Of the Several Kinds of Inter-
terminal Relations.—The relations of
terms are, for various purposes, divided in so
many different ways that it would be impracti-
cable to enumerate them. But, of these divi-
sions, there are three that, either on account of
their intrinsic importance, or of the importance
attributed to them by logicians, will require
our attention. These consist in the distinction
made (1) between the Predicables and the Cate-
gories or Predicaments; (2) between the formal
and the material relations of terms; and (3) be-
tween the relations that are intuitively per-
ceived, and those that are not, or, more briefly,
between judgments and assumptions (§ 19, 20).

§ 66. (1) Of the Predicables and of the
Otherwise we would fall into the same fallacy as Jevons and
Hobbes (v., infra, § 90 and note).
CATEGORIES OR PREDICAMENTS.—The distinction between these corresponds precisely to the distinction we have made between the *logical* and the *grammatical* forms of the proposition. Etymologically both terms are of the same import,—denoting simply terms that may be predicated of other terms, *i.e.*, that may be made *predicates* of propositions; but, according to inveterate use, the former term relates exclusively to the logical proposition, the latter, to the grammatical. There is, therefore, an essential difference between the Doctrine of the predicables and that of the Categories or predicaments. The former—which treats simply of the relation of *species and genus* between the terms expressed in the logical proposition—has already been considered. The latter treats of all the various relations that may exist between the terms of the grammatical proposition; and, as these include all relations, whatever, that may exist between terms, or between their significates, it follows that the categories or predicaments are to be understood as denoting the most general classes into which such relations may be distributed. By such a classification—if it could be accomplished—all relations between terms and between things would be developed, and thus a basis furnished for a classification of all possible predicates. But the subject is one of difficulty, and in the
present state of philosophy, a satisfactory treatment of it is impracticable. It would simply serve, therefore, to confuse the student, if we should enter upon it, and we will accordingly omit it.

§ 67. (2) Of the Formal and of the Material Relations of Terms.—By the formal relations of terms are meant those relations that are universal in their nature,—i.e., that exist generally with reference to all terms; as, e.g., the relation between terms and their contradictories, between a term used universally and the same term used particularly, between the subject and the predicate of the proposition, etc. These are all apparent at once from the mere expression, without taking note of the matter of the term, except in so far as it is universal or common to all terms. Thus, e.g., in the expression “not-man” we perceive at once a formal relation between this term and “man,” and in this case the privative “not,” though part of the matter of the term not-man, is the ground of the relation; which is formal because universal. And so, in the terms “Y” and “some Y,” a formal relation is apparent, though the word “some” is in fact part of the matter of the term “some Y.”

Hence, the distinction between the formal and the material relations of terms does not, as is commonly supposed, rest upon the
distinction that, in the former case, the matter of the term is not considered, and, in the latter, that it is; but on the distinction that the formal relations are based upon such part of the matter or meaning of terms as is common to all or to many terms, and with that regard to the material relations this is not the case.

Hence, logically, there is, in fact, no essential difference of nature between the two kinds of relation. For the material relations between terms are as apparent and as certain as the so-called formal relations,—as, e. g., the relations between relative terms, as "father" and "son," etc., or those between such terms as "island" and "continent," "island" and "water," "body" and "weight," "five" and "seven," "nine" and "fifteen," etc.; and they differ only in this, that these subsist only in particular cases, and not universally. Hence the notion that would restrict the functions of Logic to the merely formal relations of terms is based upon an unessential difference of nature between these and other relations, and therefore cannot be sustained.

§ 68. (3) Of Judgments and Assumptions. —Of the immediate relations between terms some—as we have seen—are self-evident, or may be intuitively perceived; others are not of this character. Where the relation between the terms of a proposition is of the former
kind, it is called a judgment; where the relation expressed is of the latter kind (if not an inference) it is called an assumption (§§ 19 et seq.). This division of propositions is based upon an essential difference of nature, and is one of fundamental importance. It will therefore require our most attentive consideration.

Logical Judgment Defined.—In the logical proposition, the only relation between the terms expressed is what we have called the significative relation,—i. e., the relation of inclusion or exclusion of one of the terms in or from the other. Hence judgment, in the logical sense, may be defined as consisting in the intuitive perception of a significative relation between two terms,—i. e., in the intuitive perception that the subject class is, or is not, included in the predicate class,—as, e. g., where, from our knowledge of the significiation of the terms, we affirm that "man is an animal," or that "fishes are denizens of the water," or that "bodies are affected by gravity," or that "fortitude is the only resource against the inevitable," or, in the Latin, "Quidquid crit superanda omnis fortuna ferendo est."

§ 69. Of the Distinction between Judgment and Assumption.—The product of this mental process—as we have seen—is
called a judgment; which may be defined as a proposition at once self-evident, and not inferred from another proposition or propositions. Hence, the opinion that "propositions are judgments expressed in words" is a departure from the logical definition of a judgment. A judgment expressed in words is a proposition, but the converse is not true. For where a proposition is based not merely upon a comparison of its terms, or upon an inference, but upon extrinsic evidence, or authority, or other grounds, the forming of an opinion is not a logical process, and the proposition, from a logical point of view, is to be regarded, not as a judgment, but merely as an assumption or hypothesis. Of this kind is the proposition that Pompey's army was defeated at Pharsalia; that Cicero was murdered by the Triumvirate; that a given policy as, e. g., protection to home industries, or the re monetization of silver, will be beneficial, etc.

§ 70. Of the Distinction between Apodictic and Dialectic (§ 23).—Hence it may be readily perceived how inadequate is the conception of Logic that would restrict its functions to merely formal inference to the exclusion of judgments; or the conception of demonstrative or apodictic reasoning that would confine it to the mathematics; or to the limited class of sciences that
rest upon intuitions, in the sense of the term used by modern metaphysicians; or that would exclude from it all reasoning originating in judgments involving empirical notions or concepts. For, logically, a judgment as to a significative relation between two terms denoting notions or concepts, of which the apprehension is empirical,—as, e. g., the judgment that "bodies are affected by gravity," that "fish live in water," that "food will assuage hunger," etc.,—is quite as self-evident as the judgment that "two and three are five," or that "sixty-four is the square of eight." In fact, the two classes of judgments are, logically, of precisely the same nature,—each being but an intuitive perception of a relation between the significations of two terms; as follows from our definition.

§ 71. No Distinction in Logic between a priori and Empirical Notions.—Logic, therefore, takes no account of the metaphysical distinction between a priori and empirical notions, but regards all judgments as intuitive. Its function is simply to determine the relations existing between the significations of terms; and if the significations of the terms compared be apprehended, and be of such nature that the relation between them can be perceived, either immediately—i. e. intuitively,—or by intermediary comparison with other
terms, the conclusion reached—which expresses merely the relation between the significations of the terms—is, so far, absolutely true.

§ 72. Of the Error that Ratiocination is Only Hypothetically True.—Hence it is an error to suppose that ratiocination is only hypothetically true, or, in other words, that Logic is not concerned with the truth of premises. In many cases this is so; but it is true in no case in which the ratiocination proceeds from judgments exclusively. For in all such cases the premises—which, as we have said, merely express significative relations between their terms—are not merely assumed, but are intuitively known to be true, and the conclusion is true, not hypothetically but absolutely.

And this is essentially the case even where the notions involved in the original judgments or premises are themselves false or unreal; for the ratiocination has for its direct object only to determine correctly the relation between the significations of the terms of the conclusion; and all that is directly asserted in the conclusion is that the signification of the terms are related as expressed; and hence, when the ratiocinative functions have been rightly performed, the conclusion must be necessarily true. But as it is necessary for
purposes of ratiocination that grammatical propositions be converted into logical, so also, for practical use or application, all logical conclusions must be reconverted into grammatical propositions, or, in other words, construed as asserting not merely the significative relation expressed, but also the truth or reality of the notions or concepts denoted by the terms; and when thus construed the conclusion cannot be regarded as being absolutely true, unless the terms express real notions. Hence, it may be said that the conclusions reached in ratiocination proceeding exclusively from judgments are, when construed grammatically, true only upon the hypothesis that the notions involved in the original judgments or premises are true or real, and hence, that such conclusions are true absolutely only as logically construed. Thus, e. g., the judgment that “all bodies are affected by gravity” is intuitive; but of the truth or reality of the notions expressed by these terms, respectively, we have no assurance but experience. And from these observations it may be perceived how, and in what sense, it is that Politics, Morality, and the Science of Human Nature generally are all to a large extent susceptible of demonstration, and to that extent apodictic in their nature (§§ 23 et seq.).
CHAPTER IV

DOCTRINE OF THE SYLLOGISM

I

RUDIMENTS OF THE DOCTRINE

§ 73. ELEMENTS OF THE SYLLOGISM.—The Syllogism consists of three propositions (§ 22): of which two are called the premises, and the other the conclusion. It has also three terms. Of these, two appear as the subject and the predicate of the conclusion, and are called, respectively, the minor and the major term. The other— which is called the middle term—is used in both premises: in the one with the major, in the other with the minor term. The premise containing the major term is called the major, and that containing the minor, the minor premise. Thus in the syllogism, "Y is X, Z is Y, . . . Z is X," Z is the minor, X the major, and Y the middle term; and the first proposition the major, and the second the minor premise.
§ 74. Analysis of the Syllogism.—The proposition is but the expression of a significative relation between its terms. Hence the premises of a syllogism are merely statements of the significative relations of the terms of the conclusion (the major and the minor) respectively with the middle term; and the conclusion the significative relation thereby inferred between its terms. The essential elements of the process consist, therefore, in the comparison of the two terms of the conclusion respectively with the third, or middle term, and in inferring a direct relation between them.

§ 75. Definition of the Syllogism.—Hence syllogistic inference may be more specifically defined as consisting in the inference of a significative relation between two terms from their known significative relations to a third term with which they are respectively compared.¹

§ 76. The Principle of the Syllogism.—The principle of the syllogism (by which is meant the principle or axiom on which depends the illative force or conclusiveness of syllogistic inference) is expressed in the Dictum of Aristotle, or, as it is technically called, the

¹ The definition in the text is taken substantially from that of De Morgan; who defines the syllogism as “the inference of the relation of two names from the relation of each of those names to a third” (Formal Log., p. 176).
Dictum de Omni et Nullo. It is variously stated by logicians, but the several forms are all, in effect, identical. Its best expression is as follows:

**DICTUM DE OMNI ET NULLO.**—“Where three terms (which we will call the *middle* and the *two extremes*) so subsist with relation to each other that the one extreme is contained in the middle, and the middle *is contained in* [or *excluded from*] the other extreme, then [as the case may be] the extreme included in the middle *will be included in* [or *excluded from*] the other extreme.” ¹ Where the predication is *affirmative* the principle is called the *Dictum de Omni*; where *negative*, the *Dictum de Nullo*.

Omitting in the form given above the words in brackets, it becomes the *Dictum de Omni*; substituting the words in brackets, marked as quoted, for the corresponding expressions, it becomes the *Dictum de Nullo*.

The two forms of the Dictum (affirmative and negative) correspond precisely to the two forms of syllogisms called *Barbara* and *Cela-rent*, ² viz.:

¹ This is substantially the form given to the *Dictum* by Aristotle, *Prior Analytics*, i., iv.

² *Forms of the Syllogism.* There are nineteen forms of valid syllogisms recognized by logicians, which are explained in the next chapter. But if we reject the use of *particular* propositions (§ 52 n.) all may be reduced to the two forms
THE SYLLOGISM

\[ Y \text{ is } X \quad \text{Y is not } X \]
\[ Z \text{ is } Y \quad Z \text{ is } Y \]
\[ \therefore Z \text{ is } X \quad \therefore Z \text{ is not } X \]

II

THE PRINCIPLE OF SUBSTITUTION

§ 77. RULES OF INFERENCE.—The following practical rules may be deduced from the Dictum:

(1) In any affirmative proposition we may always (without affecting its illative force or conclusiveness) substitute for the subject any other term denoting the same, or part of the same, significates; and for the predicate any term denoting the same significates, or a class that contains them.

Or, more briefly, we may always in the subject substitute species for genus; and in the predicate, genus for species.

(2) So, in any negative proposition, we may, without affecting its illative force, substitute for either subject or predicate any term denoting the same, or part of the same, significates.

Or, more briefly, we may always, in the negative proposition, either in the subject or the predicate, substitute species for genus.

above given, which are called Barbara and Celarent. In these forms the several terms may be represented indifferently by any letters; and the order of the propositions is immaterial. In the traditional Logic the order of the propositions is always as in the examples given in the text.
(3) To which may be added the following: In any affirmative proposition we may always substitute for the predicate any other term that denotes the same significates as the subject, or a class containing them.  

§ 78. **Equivalence of Terms Defined.**—In the above rules, it will be observed, the term substituted is not necessarily equivalent in signification to the term for which it is substituted; but it is equivalent so far as the force of the inference is concerned, or, as the lawyers say, *quoad* the argument. It may be said, therefore, briefly, that *mediate*, or syllogistic inference consists simply in substituting for the terms of propositions other terms equivalent in ratiocinative value.

§ 79. **Conversions of Propositions.**—The case of conversion of propositions seems indeed, to be an exception; for here the process seems to consist, not in the substitution of terms, but in the substitution of a new

---

1 The deduction of these rules from the Dictum is perhaps sufficiently obvious, but as it may not be apparent to all, we subjoin the demonstration:

In the first syllogism (*Barbara*) it will be perceived, as expressed in the *minor* premise, that *Z* is a *species*, and *X* the *genus*, of *Y*, and that the conclusion is arrived at by substituting for *Y*, in the *major* premise, its *species* *Z*; or, for *Y* in the *minor* premise, its *genus* *X*.

In the latter syllogism (*Celarent*) the process consists in substituting for *Y*, in the *major* premise, its *species* *Z*; and so it is obvious we may substitute for *X* in the *major* premise any
proposition containing the same terms as the original with the order of terms transposed. But the exception, in the case of negative and equational propositions, is more apparent than real; for the two forms of the proposition (i. e., the converted and the original proposition) are precisely the same in effect, and there is, in fact, neither term nor proposition substituted. For when we say "Y is not X," we equally and as explicitly say "X is not Y"—the meaning of either proposition being simply that the two classes denoted by X and Y are mutually exclusive; and so in the equational proposition (Y = X) we say, in the same breath, both that Y is equal to X, and that X is equal to Y. So, species of the genus X, as, e. g., A, B, or C, and thus conclude that "Z is not A, B, or C" (as the case may be); as may be illustrated by appropriate diagrams:

So, in the major premise in Barbara, we may substitute for X the expression YX, or any species of X containing Y, as, e. g., A, and thus conclude that Z is YX, or Z is A, as the case may be.
upon consideration, it will be found that the conversion of the (universal) affirmative proposition—\textit{i. e.}, conversion \textit{per accidens}\textemdash is not an exception to the rule, but an application of it; for the process consists simply in substituting for the predicate another term precisely equivalent to the subject in signification, as, \textit{e. g.}, in the proposition "$Y$ is $X$," the expression "some $X$" for "$X$,"—meaning, by the expression "some $X$," that part of $X$ which coincides with $Y$; which is but an application of Rule 3. And when this substitution is made, the proposition becomes equational, and means the same thing whether we convert it or not.

\textsection{80. Of Immediate Inferences Generally.}\textemdash Propositions derived from other propositions by conversion, and also those derived by opposition (explained \textit{infra}, \textsection{89), are regarded by recent logicians as inferences, and to distinguish them from \textit{syllogistic} inferences are called \textit{immediate}. This innovation we regard as unfortunate, though of too general use to be neglected, for, according to our view, only one kind of inference is allowed, namely, \textit{syllogistic}. This, as we have shown, includes the case of conversion \textit{per accidens}; and it also includes other, and perhaps all, cases of so-called immediate inference; as may be readily shown.
(1) **Substitution of Contradictory.**—One of these is what is called by Bishop Thompson, "Immediate Inference by Means of *Privative Conceptions*," and by other logicians, improperly, "Infinition." It is, in fact, identical with the process treated hereafter under the head of "Conversion by Contraposition" (§ 91). It consists in substituting for the predicate its negative, or contradictory, and in changing the quality of the proposition,—i. e., making the copula of the negative proposition affirmative, or that of the affirmative proposition negative. Thus, denoting the terms by the capital letters Y and X, and their negatives or contradictories by aY and aX, the negative proposition "Y is not X" may be converted into the affirmative proposition, "Y is aX"; and similarly the affirmative proposition, "Y is X," into the negative proposition, "Y is not aX" (i. e., is not Not-X). The validity of the process, as may be illustrated by the following diagrams, rests upon the principle that any negative proposition, as, e. g., "Y is not X," may always be regarded either as denying that the class Y is included in the class X, or as affirming that it is included in the class aX, or "Not-X"; and conversely the affirmative proposition, "Y is X," may be regarded either as affirming that the class Y is included in the class X, or as
denying that it is included in the class \( aX \), or "Not-X."

But when from the affirmative proposition "Y is X" we conclude that "Y is not Not-X," there is a syllogistic inference; which, denoting the negative or contradictory of \( X \) by \( aX \), may be thus expressed:

\[
\begin{align*}
X \text{ is not } aX \quad (i. e., \text{ not Not-}X) \\
Y \text{ is } X \\
\therefore \ Y \text{ is not } aX.
\end{align*}
\]

The inference, therefore, rests upon the judgment that the term "X" is equivalent to the term "Not-\( aX \)," and consists in substituting the latter for the former. Hence the principle of inference involved may be stated generally by saying that a term is always equivalent in signification to the contradictory of its contradictory, or, as otherwise expressed, the negative of its negative; which is but a different expression of the maxim that "two negatives make an affirmative."

It is, indeed, said that the major terms in the two propositions are the same—the propositions differing only in quantity, and hence
that no third term is introduced. But this is incorrect; for the major term in the former proposition is X, and in the latter "not Not-X"; and it is a fundamental logical doctrine that no two terms are identical that differ, either in denotation or connotation, or vocal sign; and also that the very essence of ratio- cination consists in the recognition of identity of signification in terms having different connotations or vocal signs, and in the substitution of the one for the other (§§ 77 et seq.).

(2) Immediate Inference by Added Determinants, and (3) the Same by Complex Conceptions.—These kinds of supposed immediate inference were introduced into Logic by Leibnitz (Davis, Theory of Thought, p. 104). The former is stated in the proposition that the same mark may be added to both terms of a judgment; the latter, in the proposition that the two terms of a judgment may be added to the same mark. Of the former, the example given by Thompson is: "A negro is a fellow-creature," therefore, "A negro in suffering is a fellow-creature in suffering"; of the latter: "Oxygen is an element," and therefore, "The decomposition of oxygen would be the decomposition of an element." The two processes seem to be in substance the same, and both may be expressed symbolically by saying that "If Y is X," then "ZY will be ZX," or (what
is the same) "YZ will be YX"; as may be thus symbolically illustrated:

This process is erroneously regarded by logicians as an immediate inference; but it is, in fact, mediate, and may be stated in syllogistic form as follows:

\[
\begin{align*}
Y \text{ is } X \\
ZY \text{ is } Y \\
\therefore ZY \text{ is } X
\end{align*}
\]

The conclusion "ZY is X," fully expressed, is that ZY is that part of X with which it coincides; or, in other words, that "ZY is ZYX." But ZYX is ZX; and hence ZY is ZX.

In this case the observations made with reference to infinitation (supra) will apply a fortiori; for here a new term, "ZY," is introduced, differing from Y in denotation, in connotation, and in verbal sign.

1 But the converse is not true,—i.e., from the proposition, ZY is ZX, we cannot infer that Y is X; as will appear from the following diagram:

\[
\begin{align*}
Y & \quad ZY \\
& \quad ZX
\end{align*}
\]
It may therefore be concluded, as already asserted, that all inference consists in substituting, for terms of propositions, other terms of equivalent ratiocinative value.

§ 81. Formal and Material Substitutions.—Substitution of terms may be either formal or material. The former includes all cases where the substituted term is the original term in a modified form,—as, where the elements of a complex term are arranged in a different order, as, e.g., where YX is substituted for XY; or, as where the original term is qualified by some other word or words expressing a formal relation existing between the substituted term and the original,—as, e.g., where in the proposition "Y is X," we substitute for "Y" "some Y," or for "X" "not Not-X"; or, as in the example given above, where we substitute for "negro" and "fellow-creature" the terms "negro in suffering" and "fellow-creature in suffering." Material substitutions are those where a new term is substituted, as, e.g., where we substitute for a term a synonym, or species for genus, or genus for species.

III

OF MATHEMATICAL REASONING

§ 82. Mathematics the Type of All Ratiocination.—Hence it would seem that
the most perfect type of ratiocination is presented by the mathematical, and especially by the algebraic methods of demonstration; and this is, in fact, the case, as may be illustrated by two familiar examples:

1st Example. Thesis.—The angles of a plain triangle are together equal to two right angles; or, referring to the figure, \(a + b + c = \Delta\) (Euclid, Book I., Prop. XXXII.).

\[
\begin{align*}
\angle A + \angle B + \angle C &= \pi \\
(\text{Euclid, Book I., Prop. XXXII.})
\end{align*}
\]

For
\[a + b' + c' = \Delta \quad \text{(Ib., Prop. XXIX.)}\]
But
\[b' = b \quad c' = c.\]

Hence, substituting equivalents,
\[a + b' + c' = a + b + c = \Delta. \quad Q. \ E. \ D.\]

2d Example. Thesis.—The formula for compound interest, \emph{i.e.}, \(S = p (1 + r)^n\), in which \(p\) = principal, \(n\) = number of years, \(r\) = rate of interest, and \(S\) = the amount.

At end of first year
\[S = p + pr = p (1 + r).\]

At end of second year
\[S = p (1 + r) + pr (1 + r) = p (1 + r)^2.\]
At end of third year

\[ S = p (1 + r)^2 + pr (1 + r)^2 = p (1 + r)^3. \]

At the end of \( n \) years

\[ S = p (1 + r)^n. \]

§ 83. A Current Error on this Point. — It is indeed asserted by recent logicians that there is an essential difference between ordinary and mathematical, or, as it is otherwise expressed, between qualitative and quantitative reasoning. But this opinion arises from the failure to reflect that the comparison of magnitudes can be effected only by means of units of measurement that can be applied equally to the magnitudes compared, and that these constitute the significates denoted by mathematical terms. Hence mathematical reasoning consists not in directly comparing the magnitudes considered, but in comparing the units that represent them; and mathematical terms must therefore be regarded as denoting — like other terms — collections or classes of individuals, i.e., of the units expressed.

An Opinion of Mr. Bain.—On this point we have the following from Mr. Bain: “Logicians are aware that the form ‘A equals B, B equals C, therefore A equals C’ is not reducible to the syllogism. So with relation to ‘greater
than' in the argument *a fortiori*; yet to the ordinary mind these inferences are as natural, as forcible, and as prompt as the syllogistic inference." But the first expression is a perfect syllogism differing from the ordinary form only in the different interpretation given to the copula; and this is true also of the argument *a fortiori*, if we give it the form, "*A < B, B < C*: ∴ *A < C.*" It is strange this is not recognized by the author; or, rather, would be strange were not the error common. What is meant, therefore, is that the mathematical cannot be reduced to the ordinary form of the syllogism. But this is not the case, for mathematical reasoning can readily be expressed in the ordinary logical forms, as, *e. g.*, the equational syllogism in the two syllogisms following:

```
<table>
<thead>
<tr>
<th>a is b</th>
<th>b is a</th>
</tr>
</thead>
<tbody>
<tr>
<td>b is c</td>
<td>c is b</td>
</tr>
<tr>
<td>∴ a is c</td>
<td>∴ c is a;</td>
</tr>
</tbody>
</table>
```

and the argument *a fortiori* in the following: "*a is b, b is c, ∴ a is c,*"—meaning that the class of units denoted by *a* is contained in the class denoted by *b*, etc.

Or the *inequalities* may be converted into equations, as, *e. g.*, "*a < b*" into "*a + x = b,*" and the argument then be expressed in two syllogisms as above.
§ 84. Reduction of Euclid’s Fifth Proposition to Syllogisms.—Recognizing the mathematical form of the syllogism, there is no need of the cumbersome method usually adopted for the reduction of mathematical reasoning to syllogistic form, as, e. g., in the ancient example of the reduction of Euclid’s Fifth Proposition given by Mansel in his notes to Aldrich; or the reduction of the same proposition by Mill (Logic, p. 142).

In fact, Euclid’s demonstration is itself in syllogistic form, and needs only a slight variation in the statement of it to make this apparent, as, e. g., as follows:

Prop. V. The angles at the base of an isosceles triangle are equal to one another.

Or, referring to the figure, in the isosceles triangle A B C the angles a and c are equal.

The figure is constructed by producing the equal sides A B and A C to D and E, making the lines A D and A E equal, and by drawing the lines B E and D C.

Demonstration

1st syllogism

Major Premise. — Prop. IV.

Minor Premise. — The triangles A B E and
A C D are triangles having two sides of the one equal to two sides of the other, each to each, and the included angle equal.

Conclusion.—They are therefore equal in all their corresponding parts, and hence B E = C D and the angle d = the angle e.

2D SYLLOGISM

Major Premise. — Prop. IV.
Minor Premise. — The triangles C B E and B C D are triangles having two sides of the one equal to two sides of the other, each to each, and the included angle equal.

Conclusion.—They are therefore equal in all their corresponding parts, and hence the angle f = the angle g.

3D SYLLOGISM

Major Premise, a = d − f. (Judgment, or intuitive proposition.)
Minor Premise, d − f = e − g.
Conclusion, a = e − g.

4TH SYLLOGISM

Major Premise, a = e − g.
Minor Premise, e − g = c.
Conclusion, a = c.
CHAPTER V
SUMMARY OF THE TRADITIONAL LOGIC

I

OF THE TRADITIONAL LOGIC GENERALLY

§ 85. As explained in the preface, one of the principal objects of this work is to vindicate, as against modern innovations, the old or traditional Logic; and accordingly, in all that has been said—with exceptions to be noted presently—I have kept close to the traditional view, as expounded by Aristotle and the most approved of the older logicians. I have, indeed, repudiated the doctrine advocated by Whately, and by modern logicians generally, that would distinguish between the formal and the material relations of terms, and restrict the scope of Logic to the former; but in this also I follow Aristotle and the better authorities.

The only particulars, therefore, in which I have departed from the traditional view of Logic are: (1) that I reject the "Particular Propositions" of the old Logic and those parts of the old doctrine of the Proposition and of
the Syllogism that are founded on this view of the proposition; and (2) that I have adopted, in place of the Dictum, the Principle of Substitution; which is an obvious corollary from the Dictum, and is more readily understood and applied.

At the same time, it must be admitted, the old doctrines of the Proposition and the Syllogism are remarkable for the accurate analysis upon which they rest, and the wonderful ingenuity and acuteness with which they have been developed. They have thus become part of the accepted philosophy of the world; and there has thus been developed a technical language that has come to be universally received and so generally used that, without an understanding of it, all the literature on the subject must be a closed book to us. I now propose, therefore, to give a brief exposition of these doctrines.

II

THE TRADITIONAL DOCTRINE OF THE PROPOSITION

§ 86. QUALITY OF PROPOSITIONS.—Propositions are said to differ in quality accordingly as they are affirmative or negative. Thus the propositions "All Y is X" and "Some Y is X" are affirmative; the propositions "No Y is X" and "Some Y is not X," negative.

§ 87. QUANTITY OF PROPOSITIONS.—Again, propositions, whether affirmative or negative,
are said to differ in quantity accordingly as the predicate is asserted, or denied universally of all individuals of the class denoted by the subject or only part of such individuals. In the former case the subject is said to be distributed, and the proposition is called universal; in the latter, the subject is undistributed, and the proposition is said to be particular. Thus, e.g., the propositions, "All Y is X" and "No Y is X" are both universal; and the propositions, "Some Y is X" and "Some Y is not X," both particular.

§ 88. Table of Propositions.—Hence, four forms of propositions are recognized by the old logicians, viz.: (1) the Universal Affirmative; (2) the Universal Negative; (3) the Particular Affirmative; and (4) the Particular Negative; which are designated respectively by the letters A, E, I, and O; and, with their expressions in Euler's Symbols, are as follows, viz.:

A: Y is X (i.e., All Y is X)

E: Y is not X (i.e., No Y is X) ¹

I: Some Y is X

O: Some Y is not X

¹ The above differs somewhat from the ordinary notation;
In the negative propositions, E and O, it will be observed, the predicate is distributed or taken universally; in the affirmative propositions it is undistributed.

§ 89. Opposition of Propositions.—Two propositions are said to be opposed to each other when, having the same subject and predicate, they differ in quantity or quality, or both.

Propositions that differ both in quality and quantity, as A and O, or E and I, are called contradictories, as, e. g., "Y is X," and "Some Y is not X"; or "Y is not X" and "Some Y is X." Those that differ in quality only, if according to which it is thought necessary in A and E to use the signs "All" and "No," in order to indicate that the subject is distributed, as, e. g., "All Y is X," "No Y is X." But, properly speaking, the signs "All" and "No" are unnecessary and redundant. For when we say, e. g., "Man is mortal," or "Man is not mortal," we mean, when we speak properly, that in the former case the class "man" is wholly included in, and in the latter that it is wholly excluded from, the class "mortal"; or, in other words, as the case may be, that "All men are mortal," or that "No man is mortal" (§ 53, n.). The last expression is also objectionable on account of the liability to confound the expression "No man" with the term "Not-man" in converting either of the above propositions by contraposition (for which see infra, § 91); or (more generally) the negative proposition "No Y is X" is liable to be confounded with the affirmative proposition, "Not-Y is X." Hence it will be preferable to regard the subject as always distributed, except where it is preceded by the adjective "some"; and, in place of the sign "no" before the subject, to use the particle "not" after the copula.
universal, are called contraries, as, e. g., "Y is X" and "Y is not X"; and if particular, sub-contraries, as, e. g., "Some Y is X" and "Some Y is not X." Where propositions differ in quantity only, as A and I, or E and O, the particular propositions are called subalterns, as, e. g., "Y is X" and "Some Y is X"; and "Y is not X" and "Some Y is not X."

There are, therefore, four kinds of opposition recognized by logicians, viz.: (1) the opposition of contradictories; (2) that of contraries; (3) that of subcontraries, and (4) that of subalterns to their corresponding universals; which, with their relations to each other, are admirably expressed in the following table, which has come to us from ancient times:

(1) CONTRADICTORIES. The most complete kind of opposition is that of contradictories. These cannot both be either true or false: i. e., if one is true, the other is false; or, if one is
false, the other is true. For if it be true that "All men are sinners," it cannot be true that "Some men are not sinners"; and, conversely, if it be true that "Some are not righteous," it cannot be true that "All men are righteous." In other words, between contradictories there is no intermediate proposition conceivable; one must be true and the other false. This is called the law of Excluded Middle.

(2) Contraries. Contraries cannot both be true; for if it be true that "Every man is an animal," it must be false that "No man is an animal." But both may be false, as, for example, the propositions that "All men are learned," and that "No men are learned"; which are both false, for some are learned and some are not. In other words, contrary propositions do not exclude the truth of either of the particular propositions between the same terms.

(3) Subcontraries. Subcontraries are contrasted with contraries by the principle that they may be both true, but cannot both be false. Thus it may be true that "Some men are just," and also that "Some men are not just"; but if it be false that "Some men are just," it must be true that "No man is just," —which is the contradictory,—and, a fortiori, that "Some men are not just," —which is the subcontrary.

(4) Subalternate Opposition. With re-
gard to subaltern propositions, their truth follows from the corresponding universal propositions; for if "all men are animals," "some men are animals," and if "no man is an ape," "some men are not apes." But from the truth of a subaltern proposition we cannot infer the truth of the corresponding universal, as, e.g., from the proposition "Some men are false," the proposition "All men are false"; or from the proposition "Some men are not false," the proposition that "No man is false."

§ 90. Observations upon Contrary and Contradictory Oppositions.—Accurately speaking, these constitute the only kinds of opposition. Subcontraries are, in fact, not opposites; and the same is true of subalterns and their corresponding universals.

It will be observed it does not follow from the principle of contrary opposition that of two terms regarded as subject and predicate—as, e.g., Y and X—either the latter or its negative may always be predicated of the former, or, in other words, that Y must be either X, or not X; for, in fact, some Y may be X, and some Y not X, as will obviously appear from the following diagrams:

![Diagrams](image-url)
Hence there arises, seemingly, a puzzling contradiction between this principle and the law of Excluded Middle—as it is often stated. Thus, it is said, "Rock must be either hard or not hard" (Jevons, *Lessons in Logic*, p. 119), or, generally, "Y is either X or not X." But obviously this, unless accidentally, is not true; for some rock may be hard and some soft; or some Y may be X, and some not X. And so we cannot say of "men" either that they are learned or that they are not learned; for some are the one and some the other. But the apparent contradiction arises from the misstatement of the law of Excluded Middle; which is itself nothing more or less than the principle governing contradictories, as expressed above. We may, indeed, where a subject term (as, *e.g.*, Y) denotes an individual or single thing (real or fictitious), affirm of it that it is either X or not X; but if Y denotes a class of more than one we cannot so affirm.1

1 Even Hobbes falls into the error of Jevons on this point. "Positive and negative terms," he says, "are contradictory to one another, so that they cannot both be the name of the same thing. Besides, of contradictory names, one is the name of anything whatsoever (*i.e.*, of any conceivable thing), for whatsoever is, is either a man, or not a man, white, or not white, and so of the rest." But, it may be asked, "Does the name 'biped' denote (universally) either man, or not man?" or "the name 'man', either white man, or man not white?"

The confusion results from the technical view that regards
§ 91. Conversion of Propositions.—A proposition is said to be converted when its terms are transposed, i. e., when the subject is made the predicate and the predicate the subject (§ 54). Such conversion is admissible only when illative, i. e., where the truth of the converse is implied in that of the original proposition. When such conversion can be made without otherwise changing the proposition it is called a simple conversion; otherwise, it is called a conversion per accidens. Thus A ("Y is X") cannot be converted simply, because the subject only is distributed; we therefore cannot say that "All X is Y," but only that "Some X is Y," which is called conversion per accidens. But E ("Y is not X")—as both subject and predicate are distributed—may be converted simply; or, in other words, we may say

the Particular Proposition as a form distinct from the Universal, and its source would be removed if, as elsewhere suggested, this form of the proposition should be rejected (§ 52, n.). We might then adopt, as equally accurate and profound, the remaining observation of Hobbes, that "the certainty of this axiom, namely, that of two contradictory names one is the name of anything whatsoever, the other not, is the original and foundation of all ratiocination, that is, of all philosophy" (Logic, Sec. 8), which is in accord with the view of Aristotle: "For the same thing to be present and not to be present, at the same time, in the same subject, and in the same sense, is impossible. . . . For by nature this is the first principle of all the other axioms" (Metaphysics, R. iii., chap. iii.).
that "No X is Y." So with I ("Some Y is X"),—as both subject and predicate are undistributed,—the proposition may be simply converted, *i. e.*, if "Some Y is X," then it is necessarily true that "Some X is Y."

By one or the other of these methods, *i. e.*, either *simply* or *per accidens*, all propositions of the forms A, E, and I may be converted. But O ("Some Y is not X") cannot be thus converted. Thus, *e. g.*, it cannot be inferred from the proposition "Some Greeks are not Athenians" that "Some Athenians are not Greeks." But such conversion may be effected by simply regarding the negative particle (not) as part of the predicate; by which expedient O is changed into I, and may be simply converted, as, *e. g.*, "Some Greeks are Not-Athenians"; which may be converted into the proposition "Some Not-Athenians are Greeks." So from the proposition "Some men are not learned," though we may not infer that "Some learned are not men," we may infer that "Some unlearned are men." This is called by the old logicians "*Conversion by Contraposition*," and by Whately, "*Conversion by Negation*." This method of conversion is applicable to A and E as well as O, and, as it is of very extensive use, we append a table of such conversions, taken, with some alterations, from De Morgan (*Formal Logic*, p. 67). In this table
(altering De Morgan's notation) the original terms of the proposition are denoted by the capital letters Y and X, and their contraries respectively by prefixing the Greek privative a. We append also for illustration the symbolical expressions for the several propositions:

A: "Y is X"; "Y is not aX"; "aX is not Y"; "aX is aY";

The righteous are happy
The righteous are not unhappy
The unhappy are not righteous
The unhappy are unrighteous.

E: "Y is not X"; "Y is aX"; "Some aX is Y";
"Some aX is not aY."
"X is not Y"; "X is aY"; "Some aY is X";
"Some aY is not aX";

Perfect virtue is not human
Perfect virtue is unhuman
Some unhuman virtue is perfect
Some unhuman virtue is imperfect.

Human virtue is not perfect
Human virtue is imperfect
Some imperfect virtue is human
Some imperfect virtue is not unhuman.

O: "Some Y is not X"; "Some Y is aX"; "Some aX is Y"; "Some aX is not aY";
Some possible cases are not probable
Some possible cases are not improbable
Some improbable cases are possible
Some improbable cases are not impossible.

It will be observed from the above table that a universal affirmative proposition can always be converted into another universal affirmative between the contradictories of its original terms by simply reversing the order of the terms and substituting for them their contradictories.

§ 92. Of Material Conversions.—It will be observed that the conversions of propositions treated by logicians have regard to the distinction, heretofore explained, between the formal and the material relations of terms (§ 66 (2)), and are confined exclusively to what may be called formal conversions, i.e., to cases where the equivalence of the converted and original propositions results from the formal or general relations of terms. But conversions of propositions based upon the material relations of terms are of essentially the same nature, as, e.g., where the proposition "John is the son of William" is converted into the proposition "William is the father of John"; or the proposition "Cain murdered Abel" into the proposition "Abel was murdered by Cain," or into the proposition "Cain is the man that
murdered Abel." These, having regard to the received distinction between the \textit{formal} and the \textit{material} relations of terms, may be called \textit{material} conversions; and are infinitely the more numerous class, and equally deserving of attention. But though conversions of this kind are in constant use, and though, indeed, we cannot proceed a step in our logical processes without them, yet the subject has received but little attention, and remains as yet a vast, unexplored domain.\footnote{To this domain belong such subjects as the "\textit{Categories}," "\textit{Intensive Propositions}," "\textit{Hypothetical Propositions}," and, in short, all forms of expression that differ from the ordinary logical proposition. With these Logic is concerned only in so far as is involved in their conversion into logical forms. Otherwise, neither the \textit{Intensive} nor the \textit{Hypothetical} Logic (if we may give either the name) can be regarded as part of Logic as traditionally received; which is based exclusively upon the logical form of the proposition and its \textit{extensive} interpretation. With regard to the Hypothetical Logic, it will be observed, it has no place in Aristotle's treatises; and Mansel is of the opinion—in which I agree—that in this he showed a juster notion of the scope of Logic than his successors. The subject is well treated in the current works on Logic, and is worthy of some attention from the student.} It can only be said, therefore, in the present condition of logical doctrine, that as the distinction between the \textit{formal} and the \textit{material} relations of terms has been found unessential, so must the distinction between \textit{formal} and \textit{material} conversions be regarded. Both classes of conver-
sions rest equally for their validity simply upon judgments as to the equivalence of expressions.

III

THE TRADITIONAL DOCTRINE OF THE SYLLOGISM

§ 93. The following epitome of the doctrine of the syllogism as traditionally received, brief as it is, will—with what has already been said—be found amply sufficient to expound it. It will, indeed, require the same close attention and thought as is usually given to mathematical demonstrations; but it may be said that to those who are unwilling to give, or are incapable of giving, to it this kind of thought, the study of Logic cannot be of much benefit.

1. Of the Moods and Figures of the Syllogism

§ 94. MOODS OF THE SYLLOGISM.—The syllogism is said to be in different moods, according to the occurrence and arrangement in it of the several forms of the proposition—A, E, I, and O; as, e. g., in the syllogism "Y is X, Z is Y, .:. Z is X," which consists of three universal affirmative propositions, and is, therefore, said to be in the mood A A A.

The four forms of the proposition, A, E, I, O, may be arranged, in sets of three each, in sixty-four different ways, but upon examination it is found that of these there are eleven
arrangements only that constitute valid syllogisms; and hence the legitimate syllogism can have but eleven moods, viz.:

Table of Moods


§ 95. Figures of the Syllogism.—Again, syllogisms are said to be of different figures, according to the position of the middle term in the syllogism with reference to the extremes; and as there are said to be four different ways in which the middle term may be thus placed, syllogisms are said to have four figures, viz.: the 1st figure, where the middle term is the subject of the major and the predicate of the minor premise; the 2d, where it is the predicate both of the major and of the minor premise; the 3d, where it is the subject of both the major and the minor premise; and the 4th, where it is the predicate of the major and the subject of the minor premise. Thus—using the conventional symbols—the forms of the different figures are usually expressed as follows:

Table of Figures

<table>
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<tr>
<th>1st Fig.</th>
<th>2d Fig.</th>
<th>3d Fig.</th>
<th>4th Fig.</th>
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<td>Y X,</td>
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If the eleven moods of the syllogism were all valid in each of the four figures, there would result forty-four different kinds of syllogisms differing in mood or figure. But none of the moods are valid in all the figures; and it is found on examination that there are in fact only twenty-four kinds of syllogisms that are valid; and that of these five are useless. So that the number of different kinds of legitimate syllogisms recognized by logicians is nineteen.

§ 96. Reduction of Syllogisms.—All these forms may, however, be reduced or converted — without affecting their validity — into the form of the first figure; which is accordingly regarded by logicians as the principal, or normal figure of the syllogism. The different figures and moods of the syllogism, and the methods of reduction or conversion from one figure to another, are briefly expressed in the following hexameter verses, constituting what may be called

The Table of Syllogisms

Fig. 1—Barbara, Celarent, Darii, Ferioque, prioris
Fig. 2—Cesare, Camestres, Festino, Fakoro, secundae
Fig. 3—Tertia, Darapti, Disamis, Datisi, Felapton, Dokamo, Feriso, habet, quarta insuper addit
Fig. 4—Bramantip, Camenes, Dimaris, Fesapo, Fresison.
In these lines the words commencing with capital letters (except "Tertia") are the names of the several syllogisms in each figure, and the italicized vowels point out the moods of the propositions constituting the several syllogisms. Thus, *e.g.*, the vowels indicate that *Barbara* consists of the three propositions, A, A, A; *Celarent* of E, A, E; *Darii* of A, I, I; *Feriso* of E, I, O, etc.

The initial letter in the name of each syllogism in the second, third, and fourth, or, as they are called, the *indirect* figures, indicates that the given syllogism is to be reduced to the syllogism in the first figure commencing with the same letter, as, *e.g.*, *Cesare, Camestres, Camenes* into *Celarent*; *Bramantip* into *Barbara*; *Darapti*, etc., and *Dimaris*, etc., into *Darii*; *Festino*, etc., *Felapton*, etc., and *Fesapo*, etc., into *Ferio*.

The letters *s*, *p*, and *k* indicate that the proposition indicated by the vowel immediately preceding is to be converted — *s* indicating simple conversion, *p* conversion *per accidens*, and *k* conversion by contraposition, or negation.¹

¹ The use of conversion by contraposition as a means of reduction is a late invention. It is, in general, used only in the two forms, *Fakoro* and *Dokamo*,—or, as they were originally called, *Baroko* and *Bokardo*,—as all other forms can be reduced without its aid, *i.e.*, by the use of simple conversion or conversion *per accidens*. Prior to the use of this method,
The letter \( m \) indicates that the premises are to be transposed.

The other letters are without significance.

**TABLE OF SYLLOGISMS.** By the use of the "Table of Moods" and the "Table of Figures," all the syllogisms given in the "Table of Syllogisms" may be readily constructed, and the mode of reducing the syllogisms in the second and third and fourth figures to the corresponding syllogisms in the first figure be readily perceived.

Baroko and Bokardo could not be directly reduced to the first figure, but indirectly only by a process called *reductio ad impossible*; which consisted in substituting for one of the premises the contradictory of the conclusion.

By this method Baroko is converted into a syllogism in *Barbara*, having the contradictory of the original conclusion for a minor premise, and the contradictory of the original minor premise for a conclusion, which, as the minor premise is true *ex hypothesi*, is an absurdity, viz.:

\[
\begin{align*}
\text{(Original Syllogism)} & \quad \text{(Reduced Syllogism)} \\
X & \text{is} \quad Y & X \text{is} \quad Y \\
\text{Some } Z \text{ is not } Y & \quad Z \text{ is } X \\
\therefore \text{ Some } Z \text{ is not } X & \quad \therefore \quad Z \text{ is } Y
\end{align*}
\]

By the same method Bokardo is converted into a syllogism in *Barbara*, having the contradictory of the original conclusion for a major premise, and the contradictory of the original major for a conclusion, *e. g.*:

\[
\begin{align*}
\text{Some } Y \text{ is not } X & \quad Z \text{ is } X \\
Y & \text{is} \quad Z & Y \text{ is } Z \\
\therefore \text{ Some } Z \text{ is not } X & \quad \therefore \quad Y \text{ is } X
\end{align*}
\]

1 A table of the several syllogisms, with their reductions, illustrated by Euler’s symbols, is appended (see Appendix M).
§ 97. Observations upon the Forms of Syllogisms.—It will be observed from what has been said that the numerous forms of syllogisms recognized by the old logicians result from two assumptions—the one erroneous and the other unnecessary.

The first is the erroneous assumption that the symbols $Y$ and $X$ must always be taken as denoting respectively the minor and the major terms; from which results that there are four figures of the syllogism, instead of three. But if in the fourth figure we regard $X$ as the minor term and $Y$ as the major, it becomes of the first figure. Hence the fourth figure—which was not recognized by Aristotle, but is a late invention—is rightly rejected by the best authorities.

The other assumption is that the particular propositions ("Some $Y$ is $X$" or "Some $Y$ is not $X$") are to be regarded as involving the same terms as the universal ("$Y$ is $X$" or "$Y$ is not $X"), and the expression "some" as a mere sign of quantity; from which (and the first assumption) there result the four forms of the proposition, $A$, $E$, $I$, and $O$, and the nineteen forms of syllogism recognized by logicians, Barbara, Celarent, etc.¹

¹ The doctrine of the syllogism, and especially that of its moods and figures, has been elaborated by the logicians perhaps to an unnecessary extent, but as it stands must always
§ 98. Proposed Simplification of Forms.—But if in the *particular* propositions (I and O) we regard the expression "some" not as a sign of quantity, but as part of the term,—or, in other words, if we regard "Some Y" instead of "Y" as the term,—they become the same as "A" and "E," *i. e., Universal* (§ 52, n.). By this simple change the four forms of the proposition are reduced to two (A and E), and the nineteen forms of syllogism to the two simple forms of *Barbara* and *Celarent*.  

2. Of the Dictum de Omni et Nullo

§ 99. Of the Several Forms of the Dictum.—The principle of the syllogism, or the *Dictum de Omni et Nullo*, has already been considered at length, and what has been said is sufficient to elucidate its nature. It is, however, variously stated by logicians, as indeed by Aristotle himself, and it will be of interest to consider some of its various forms.

constitute a necessary part of a liberal education. For practical use, however, it is unnecessarily complicated; and it will be found that when modified, as we have suggested (*i. e., by rejecting the particular proposition, and substituting for the ordinary form of the dictum the Principle of Substitution*), the simplicity of its application will be largely increased.

1 More accurately, perhaps, it should be said to *four* forms, namely, *Barbara*, *Celarent*, *Cesare*, and *Camestres*. But the last two are essentially the same as the second, and there is no advantage to be gained by distinguishing them.
Of these, in addition to the form already given, and which is on all accounts to be preferred, there are two others to which we will refer.

These, as given by Whately, are as follows:

"Whatever is predicated [*i.e., affirmed or denied*] universally of any class of things, may be predicated in like manner [*viz., affirmed or denied*] of anything comprehended in that class" (*Logic*, bk. i., § iv.).

"Whatever is predicated of a term distributed, whether affirmatively or negatively, may be predicated in like manner of everything contained under it" (*Id.*, bk. ii., chap. iii., § 2).

In effect, these two statements may be taken as types of all the other forms of the dictum. But, as we have observed, "*thing*" or "*things*" is an extremely vague and unsatisfactory term, and it would be better to substitute for it the expression "*significate,*" or "*significates.*"

These two forms of the dictum are in effect the same. For to say, as in the latter, "Whatever is predicated of a term distributed," is in effect to say, "Whatever is predicated universally of any class," etc. Bearing this in mind, and substituting "*significates*" for "*things,*" both forms of the dictum may be more briefly expressed by saying that "a term predicated of a term may be predicated also of any or all of its significates." Where the pred-
ication is affirmative the principle, as we have seen, is called the *Dictum de Omni*; where it is negative, the *Dictum de Nullo*.

It is said by Whately that the dictum "cannot be directly or immediately applied to all even categorical syllogisms, but, as all syllogisms may be reduced to the first figure, it may be ultimately applied to all." Hence, "to avoid the tediousness of reducing all syllogisms to that form to which Aristotle's dictum is applicable, it has been deemed necessary to invent separate rules or canons for the indirect figures" (Whately, *Logic*, bk. ii., chap. iii., § 2); and in this logicians generally agree.

§ 100. **Canons of the Several Figures.** —These canons of the several figures—omitting the fourth figure, which is disallowed by the best authorities as being a mere perversion of the first—are as follows:

**First Figure:** The *Dictum de Omni et Nullo*, as above.

**Second Figure:** *Dictum de Diverso*. If one term is contained in and another excluded from a third term, they are mutually excluded.

**Third Figure:** *Dictum de Exemplo*. Two terms which contain a common part, partly agree, or, if the one term contain a part which the other does not, they partially differ (Devey's *Logic*, pp. 109–111).

§ 101. **The Dictum, Rightly Expressed,**
Applicable to All the Figures.—But if the form of the dictum we have adopted, and which is substantially as given by Aristotle (§ 76), be taken, it will be found to apply to all syllogisms universally. But as the form given in the paragraph cited has reference to the division of propositions there adopted into two kinds only (namely, A and E, rejecting I and O), it must now be stated somewhat differently, so as to apply to the ordinary division of propositions into their four kinds, A, E, I, and O:

"Where three terms (which we will call the middle and two extremes) so subsist with relation to each other that the one extreme is included (wholly or partly) in the middle, and the middle is included in or excluded from the other, then (as the case may be) the extreme included in the middle will be (wholly or partly) included in or excluded from the other extreme."

Or dividing the proposition, and leaving the terms "wholly" or "partly" to be supplied as required, it may be stated thus:

Dictum de Omni: (a) If one extreme of a syllogism be included in the middle and the middle in the other extreme, then will the former be included in the latter.

Dictum de Nullo: (b) If one extreme of a syllogism be included in the middle, and the
middle be excluded from the other, then will the former extreme be excluded from the latter.

In this form the dictum may be readily applied to each of the three figures.

With regard to the first this is sufficiently obvious; for the syllogisms in this figure are in fact but mere symbolical expressions of the dictum — that is to say, Barbara and Darii of the Dictum de Omni, and Celarent and Ferio of the Dictum de Nullo.

With regard to the second figure, the Dictum de Nullo is, in effect, identical with the Dictum de Diverso. For to say, as is said in the former, that "the middle term is excluded from the last extreme," is in effect to say, "that extreme is excluded from the middle"; and hence the Dictum de Nullo agrees with the Dictum de Diverso in asserting that two terms, the one of which is included in and the other excluded from a common middle term, are mutually excluded.

So in the third figure the dictum is equally applicable. For in the affirmative forms (DARapti, Disamis, and Datisi) it is asserted that the middle is contained in, and in the negative forms (Felapton, Dokamo, and Feriso) that it is excluded from one of the extremes; and in both it is asserted, in effect, that the other extreme is partly included in the middle. Hence the former come directly under the Dictum de
Omni, and the latter under the Dictum de Nullo.

That the dictum agrees with the Dictum de Exemplo, however, cannot be said; for that, in terms, merely asserts the truism that "two terms which contain a common part" in that respect agree, or, "if one contain a part which the other does not," to that extent differ. But it gives us no information as to the principle by which it is determined whether the two terms have or have not a common part. Whereas the dictum of Aristotle explains that if one extreme be partly included in the middle, and the middle be either wholly included in or excluded from the other extreme, then the two extremes will or will not agree or have a common part, as the case may be.

It is therefore obvious that the dictum of Aristotle applies equally to all syllogisms, and that to invent separate canons for the several figures is unnecessary and productive of confusion.

§ 102. The Dictum Applicable to Singular and Other Equational Propositions.—It has also been objected to the dictum by several logicians that it is not applicable to syllogisms in which the terms are singular, or to other syllogisms composed of equational propositions; which, it is said, are governed by a different regulating principle,
viz., that "notions equivalent to one and the same third notion are equivalent to each other" (McCosh, *Logic*, pp. 126, 127). But this is obviously not so. For an individual may, for logical purposes, be regarded as a class (*i.e.*., a class of one); and classes that are equal to each other mutually include each other. Hence the dictum applies directly to syllogisms of this character; and we may always express such a syllogism, *e.g.*, $Z = Y$, $Y = X \therefore Z = X$, in the usual form: $Z$ is $Y$, $Y$ is $X \therefore Z$ is $X$.

§ 103. Of Proposed Improvements on the Dictum.—Other objections are urged to the dictum of Aristotle by modern logicians, and, to remedy its supposed defects, numerous new dicta or canons have been invented to take its place. But these will be found on examination to be either erroneous or merely different and less satisfactory statements of the old dictum.

In at least this fundamental aspect of the subject the opinion of Kant with reference to the Old Logic must be accepted, viz., that "Since Aristotle it has not had to retrace a single step, and to the present day has not been able to make one step in advance."  

1 In these views I find myself supported by the following judicious observations of Professor Jevons:

"During the last two or three years," he observes, "the
3. Rules of the Syllogism

§ 104. STATEMENT OF THE RULES.—The following rules, with the fallacies resulting from their violation, are given by logicians. They are all obvious deductions either from the definition of the syllogism or from the *dictum* of Aristotle.

(1) Every syllogism has three, and only three, terms, viz., the Major, the Minor, and the Middle term.

The violation of this rule is called the Fallacy of Four Terms (*Quarternio Terminorum*). It generally results from the ambiguity of a term, and indeed can hardly occur in any other way.

(2) Every syllogism contains three, and only three, propositions, viz., the Major and the Minor premise and the Conclusion.

This rule can be violated only by violating the first rule, and is therefore to be regarded as superfluous.

(3) The Middle term must be distributed once at least in the premises.

thought has constantly forced itself on my mind, that the modern logicians have altered the form of Aristotle's proposition without making any corresponding alterations in the *dictum* or self-evident principle, which formed the fundamental postulate of his system. Aristotle regarded the proposition as stating the inclusion of one term or class within another; and his axiom was perfectly adapted to this view (*Pure Logic*, p. 86).
The violation of this rule is called the Fallacy of Undistributed Middle, as, *e. g.*, in the following pseudo-syllogism: X is Y, Z is Y. ∴ Z is X.

(4) No term must be distributed in the conclusion that was not distributed in one of the premises.

The violation of this rule is called the Fallacy of the Illicit Process of the Major or of the Minor term, as the case may be, as, *e. g.*, in the following syllogism: Y is not X, some Z is Y. ∴ Z is not X,—Nations capable of self-government should not be despotically governed; some nations are capable of self-government; no nation should be despotically governed,—which is a case of *illicit process of the Minor term*; or as in the following syllogism: Y is X, Z is not Y. ∴ Z is not X,—Anglo-Saxons love liberty, Frenchmen are not Anglo-Saxons. ∴ Frenchmen do not love liberty,—which is an *illicit process* of the Major.

(5) From negative premises nothing can be inferred.

The violation of this rule is called the Fallacy of Negative Premises; *e. g.*, Y is not X, Z is not Y. ∴ Z is X or Z is not X.

(6) If one premise be negative the conclusion must be negative; and, *vice versa*, to prove a negative conclusion one of the premises must be negative.
The violation of this rule may be called the Fallacy of Affirmative Conclusion, *e.g.*, \( Y \) is \( X \), \( Z \) is not \( Y \) \( . \) \( . \). \( Z \) is \( X \).

And from the above rules may be deduced, as corollaries, the following:

(7) From two particular premises no conclusion can be drawn.

(8) If one premise be particular, the conclusion must be particular.

4. Of Enthymemes and Sorites

§ 105. Of Enthymemes.—An Enthymeme is a syllogism incompletely stated, but in which the omitted parts are understood or implied. Most commonly the omitted part is the major premise, which is then said to be suppressed, as, *e.g.*, "Caesar was a tyrant, therefore he deserved death," where the suppressed premise is the major, "All tyrants deserve death." Or the suppressed premise may be the minor, as, *e.g.*, "Freemen are happy, therefore the English are happy," where the suppressed premise is the minor, "Englishmen are freemen."

§ 106. Of Sorites.—The Sorites consists of a string of syllogisms in the first figure, in which the conclusion of each is made the premise of the next, and so on, till finally in the conclusion the predicate of the last premise
LOGIC

is predicated of the subject of the first, as, *e. g.*, A is B, B is C, C is D, D is E. \( \therefore \) A is E; or, to give a concrete example, "The English are brave, the brave are free, the free are happy, therefore the English are happy." Obviously a Sorites may always be resolved into as many separate syllogisms as it has middle terms, as, *e. g.*, in the above example, the first into three and the last into two syllogisms, as follows:

\[
\begin{align*}
A & \text{ is } B \\
B & \text{ is } C \\
\therefore & A \text{ is } C
\end{align*} \quad 
\begin{align*}
A & \text{ is } C \\
C & \text{ is } D \\
\therefore & A \text{ is } D
\end{align*} \quad 
\begin{align*}
A & \text{ is } D \\
D & \text{ is } E \\
\therefore & A \text{ is } E
\end{align*}
\]

The English are brave \quad \text{The English are free}
The brave are free \quad \text{The free are happy}
\( \therefore \text{ The English are free} \quad \therefore \text{ The English are happy.} \)
BOOK II

APPLIED LOGIC
§ 107. Of the Method of Logic.—The logical processes, as we have hitherto considered them, consist in three operations, namely, Simple Apprehension, Judgment, and Syllogism or Inference; of which the first is an analytical process, the second and third synthetical. Hence the logical processes may be regarded as twofold, and as consisting in Analysis and Synthesis. The first of these, however, is not confined to Simple Apprehen-
sion or analysis of terms, but extends to the analysis of propositions and syllogisms, and of extended discourse; of which the elements are syllogisms. It also extends, as preparatory to the expression in logical form of subjects to be investigated, to the analysis of the general facts involved and the determination of the questions to be investigated. The logical method consists in the use of these processes.

§ 108. LOGICAL DISTINGUISHED FROM PHYSICAL ANALYSIS AND SYNTHESIS.—The terms analysis and synthesis are used in different senses, according to the subject-matter to which they are applied. Of these, two principal kinds may be distinguished, which may be called, respectively, physical and logical—the former dealing with physical substances, the latter with notions or concepts. Of the former kind, the most instructive illustration is presented by chemistry; where these processes are applied directly to matter, which is analyzed by separating its elements, and synthesized by rearranging those elements so as to form new compound substances. These processes are indeed essentially different in nature from the processes with which we are now concerned, yet the analogy between the two is almost perfect; and hence, in chemical analysis and synthesis, we find the best illustration of the nature of analysis and synthesis of notions or
terms, by which—in a way very similar to the analysis and synthesis of material bodies—
notions are analyzed into elementary notions, and these again synthesized into compound.

§ 109. Of the World of Things and the World of Thought.—The world of things is made up of actual things or substances; the world of thought, of concepts or notions. There is between the two a regular correspondence, i. e., a correspondence determined by invariable law, and yet the two are clearly distinct. For it is obvious that things themselves cannot be in the mind but only, notions or concepts of them. These, as we have seen, if real or true, must correspond, either directly or indirectly, with the things which, or the attributes of which, they are supposed to denote (§ 29, n.). Where the correspondence is indirect, the thing denoted is a quasi-thing only, and cannot be distinguished from the notion itself; but where the correspondence is direct, there is a real thing corresponding to the notion, and we may either regard the notion or the thing as the significate of the term (§ 37, n.); though even in this case it is really the notion, not the thing, that we have in mind (§ 38, n.). So that it may be said that Logic, and science generally, deal directly with concepts or notions only—that is to say, with the world of
thought only, and with the world of real things only indirectly.

§ 110. Logic as the Doctrine of Signs (Semeiōtike).—But the notions or thoughts dealt with by Logic are not the evanescent thoughts of the individual, but the common notions of mankind embodied in language (§ 30). Hence, as we have observed, Logic is exclusively conversant with language, or rather, more specifically, with terms and their various ratiocinative combinations (§§ 14, 16)—that is to say, with the signs of the notions or concepts and of their relations; which cannot be dealt with, at least to any considerable extent, except by means of the vocables by which they are signified. Hence Logic must be regarded, in its direct scope, as dealing with the signs by which notions and their relations are expressed—precisely as, in the mathematics, the subject-matter dealt with consists of the signs of numbers and of their relations. In both cases, therefore, though the ultimate object of Logic is to determine the notions expressed in terms and their relations, and ultimately the nature and the relations of the things corresponding to the notions, yet this is effected by means of signs, which, therefore, constitute the immediate subject dealt with. Hence Locke was quite right in conceiving that a science of this char-
acter is indispensable, and in giving it the appropriate name of "Semeiotike, or the Doctrine of Signs," though quite wrong in supposing that this would be a new kind of Logic.

§ 111. **THE METHOD OF LOGIC RESUMED.**
—By the method of Logic is meant the method of its use in reasoning, or, in other words, the method of ratiocination, or explicit reasoning. This, as we have said, consists in two processes or operations, namely, Analysis and Synthesis, i. e., of language (§ 107). By analysis is meant the separation of a whole—whether consisting of a term, proposition, syllogism, or larger discourse, or of the general problem or subject to be investigated—into its component parts; by synthesis, the comparison (or placing together) of any of the elements of reasoning, with a view to determining their relations; that is to say, in the comparison of terms, in order to form a judgment of their relations—of propositions, in order to make an inference; and of syllogisms, in order to make an extended ratiocination or argument. Analysis and synthesis are, therefore, each the converse of the other.

§ 112. **MODES OF APPLICATION OF THE**

1 "The consideration then of ideas and words, as the great instruments of knowledge. . . . Perhaps if they were distinctly weighed and duly considered, they would afford us another sort of Logic and critic than what we have hitherto been acquainted with" (see Appendix N).
LOGICAL PROCESSES.—In each stage of ratio-
cication analysis and synthesis are used con-
jointly, and each is equally indispensable. The
order in which their applications occur, how-
ever, differs according to the purpose we have
principally in view, which may be either In-
vention or Criticism; that is to say, either (1)
the Discovery of Truth, or (2) the Criticism or
Judgment of what is supposed or alleged to be
true; or, in other words, the verification of
truth and the detection of fallacy. Of these
two aspects of Logic, the latter is commonly,
and perhaps rightly, regarded as the more im-
portant, or, at least, as of the greater practical
utility. But the former, though commonly
undervalued, is hardly of less utility or less
fruitful of practical results.

§ 113 (1) INVENTION.—The operations of
Logic, regarded as an Instrument or Organon
of Invention, consist in the analysis and conse-
quent apprehension of terms (Simple Appre-
hension), and in the discovery or invention of
judgments and of syllogisms, and of argu-
ments—which are composed of syllogisms;
which is effected by synthesis; and the process
of ratiocination proceeds in this order, i. e.,
from the term to the proposition, from the
proposition to the single syllogism, and from
that to the extended discourse or argument.

§ 114. OF THE DISTINCTION BETWEEN
Original and Commonplace Thought.—Where the notions expressed in terms are distinctly apprehended, and, with reference to all terms, to the extent they are apprehended, the relations between them are readily perceived, and indeed spontaneously present themselves. Hence with such notions men reason with facility and accuracy; and thus originate the numerous opinions that are common to mankind, or common at least to men generally under the same conditions of environment; and also those that are common to large classes of men. Of such opinions—which may be appropriately named Commonplace—the current literature and thought of the day largely, or, we may say almost exclusively, consist. Hence the effect of current thought and discourse is simply to disseminate such opinion more widely, and thus gradually to develop and consolidate Common Opinion, or Conscience, which has been called by the Greeks Nomos, and by some philosophers Common Sense. This, indeed, is a useful and essentially necessary function; for it is recognized by political writers generally that opinion is the ultimately controlling force in politics, and that when it becomes universal and inveterate, it is supreme. But current thought is marked by an essential characteristic, or, we may say, defect—namely, that it is incompatible with originality, either
in the acquisition of new truths or in the appreciation of original thought in others. Hence it has happened, throughout the history of mankind, that the results of original thought meet with almost insuperable obstacles to their reception; and that, even where they have established their footing, they pass into the hands of commonplace thinkers, who treat them after their own methods. Hence the original works of great thinkers, with their methods of thought and expression, and the vivifying effect of actual example, are submerged by the newer and inferior literature.

On the other hand, where the Analytical Method is rigorously applied to all forms of discourse, and especially when it is applied to the notions or concepts embodied in terms, numerous delicate and important but unsuspected relations between the notions thus determined suggest themselves. For in this also logical is like chemical analysis, where, by the resolution of compound substances, thousands of relations between them and between the elements of which they are composed are developed and disclosed. The perception of these unsuspected relations constitutes originality, which is but another name for logical power. Nor is this originality anywhere more conspicuously displayed than where men of original genius, as, e. g., Bacon
in his *Essays*, deal with commonplace subjects.¹ Hence the use of Logic as an Instrument of Invention cannot be too highly appreciated, for in the capacity to use Logic in this way, or, in other words, in the capacity to apprehend the whole significance of terms by resolving them into their elements, lies the essential difference between the Original and the commonplace Thinker.¹

§ 115 (2) CRITICISM.—In this aspect Logic may be likened to the touch of Ithuriel’s spear.²

¹ Where terms are clearly defined and analyzed into their constituent elements,—that is to say, thoroughly apprehended,—innumerable relations between them are intuitively perceived; and thus, by the use of this method, we are led on, as Locke says in a passage cited (*supra* § 6, n.), “from very plain and easy beginnings, by gentle degrees and a continued chain of reasonings, . . . to the discovery and demonstration of truths that appear, at first sight, beyond human capacity.” This it was, probably, that inspired the beautiful hymn of Newman:

“Lead on, Heavenly Light; amid the encircling gloom, Lead Thou me on”;

which may be very properly regarded as in reality an ode to the divine gift of Intuition—the only source of perfect knowledge.

² “Him there they found
Squat like a toad at the ear of Eve.

Him thus intent Ithuriel with his spear
Touched lightly; for no falsehood can endure
Touch of celestial spear, but returns
Of force to its own likeness; . . .
So started up in his own shape the fiend.”
Commonly the reasoning processes operate unconsciously and automatically, and the reasoning is more or less inaccurate, and hardly ever consecutive or logically coherent. As observed in the Introduction, proposition follows proposition in our minds, suggested by various principles of association, such, *e.g.*, as experience, habit, authority, inclination, etc.; and thus the great mass of our opinions and beliefs—which we very erroneously call our knowledge—comes to us we know not how. Nor, however firmly we may be convinced of them, or however passionately we may assert them, have we any just assurance of their truth; nay, it is matter of familiar knowledge that they are all mingled with error. Hence, we concluded, the necessity is apparent for some test or criterion by which to judge them; and this, except the sometimes painful test of experience, can be nothing else than Logic. In its critical aspect, therefore, Logic is indispensable, not only to save us from errors and absurdities, but to distinguish real from unreal knowledge, and to give us assurance of the former (§ 7 *et seq.*). Without it, except in concrete matters, no man can know whether he is right or wrong; and while some, happily born, learn by practice the application and use of the logical processes, the great mass of mankind, for the lack of Logic, go through life
mistaking falsehood and even nonsense for knowledge, and yet firmly convinced of their wisdom and of the folly of those who differ from them. Hence, in the critical aspect of Logic, the order of applying the logical processes is the reverse of what it is in the use of Logic as an organon or instrument of invention. There the order is to commence with the analysis of the term, and then to proceed to the synthesis of terms in propositions, syllogisms, and extended discourse; here we commence with the complex result, and by analysis resolve it into its elements.

§ 116. Of the Use of Analysis Generally.—In the use of Logic, whether for invention or for criticism, analysis and synthesis are equally indispensable; but the latter, after the former has been effected, is largely a natural and spontaneous process, and presents but little difficulty in its performance. On the other hand, analysis, while to a certain extent also spontaneous, requires, for its efficient performance, the most vigorous and protracted exertion of the mental faculties,—as, e.g., in the mathematics,—and hence is at once the most important and the most difficult of the logical processes. It will therefore require our special attention.

We have distinguished between the invention and the critical functions of Logic, and
also with reference to the use of the logical processes as applied in the performance of the one or the other function; and with reference to invention, we have regarded the function of analysis as limited to the analysis of terms, with a view to an apprehension of the notions expressed by them. In practice, however, it is difficult to distinguish between the uses of analysis for invention and for criticism. For, as we have observed, the human mind is so constituted that the synthetical process is performed spontaneously and involuntarily. Hence there is no subject that can present itself for our investigation which we can approach unembarrassed by opinions already formed; and, indeed, until such opinions or theories are formed, the process of investigation cannot commence. Hence, as is generally recognized, the method of scientific investigation must consist largely in the forming of theories and their subsequent investigation. We may distinguish, however, between our own theories, either accidentally formed or formed for the purpose of the investigation of a proposed subject, and the theories formally propounded by others, either in writing or speech; and we may conveniently regard the former as belonging to the function of invention, and the latter to that of criticism. The latter, as being the simpler subject, will be first considered.
§ 117. (1) Of the Use of Analysis in Criticism.—In this case the function of analysis extends to the analysis of all forms of language, from the term to the extended discourse or argument; and, as we have observed, it commences with the latter, which is in fact the most difficult task. For here it is necessary to determine from the loose and inaccurate expressions of ordinary disquisition the precise nature of the conclusions asserted and of the arguments used to establish them; and this task is always difficult, and sometimes impossible. When these matters have been determined it will be necessary also to analyze carefully every syllogism, proposition, or term involved in the course of the reasoning. But this in general, to the trained logician, presents but little difficulty.

§ 118. (2) Of the Use of Analysis in Invention.—Strictly speaking, this perhaps extends only to the analysis of the term with a view to simple apprehension, and in a previous passage we have so regarded it. But before this task can be approached, it is necessary for us to determine the nature of the precise questions to be investigated; and this will require an analysis of the facts involved in the investigation, and also of the opinions or theories with regard to those facts casually existing in the mind. For, as will be explained more fully in
the next chapter, the questions demanding investigation are in general determined by the nature and the conditions of the problems involved; and it is essential to a rational investigation that the issues thus involved be clearly ascertained. When the issues or questions are thus determined and logically expressed, our investigation is then narrowed to the determination of the truth of one of two alternative propositions, which are called the thesis and the anti-thesis, and of which one or the other must be true; and thus our task is in general greatly facilitated. The use of this sort of analysis finds its best illustration in the practice of the lawyers, with whom it is an imperative rule that the first step in the investigation of a case must consist in settling the issues. In ordinary discourse this task is almost always neglected, and, as will be seen as we proceed, this is one of the most fruitful sources of fallacy.

§ 119. Of Analysis and Synthesis Generally.—This subject is one of extreme importance, and to the advanced student should constitute one of the principal subjects for his meditations; but for the purposes we have in view it may be sufficiently developed by a statement of the practical rules by which the reasoner should be governed, which will be given at length in the next chapter.
CHAPTER VII

THE RULES OF LOGIC

I

OF THE RULES OF LOGIC GENERALLY

§ 120. Scope of the Rules of Logic.—According to the view we have taken in this essay, inference is only one of the processes of ratiocination. Judgment is also a ratiocinative process, and, like inference, must have its rules by which false or pretended judgments may be distinguished from the real. Moreover, where our reasoning is not apodictic, we have to use assumed propositions, or assumptions, as premises; and though it is said that Logic is not concerned with the truth or falsity of these, yet this is true only in a qualified sense. For where the falsity of such propositions can be detected by logical processes,—i. e., by definition, judgment, and inference,—it is the function of Logic to condemn and reject them; precisely as in the case of self-contradictory
propositions or propositions otherwise absurd on their face. And in all cases it is its function to determine the logical character of an assumed premise, as being an assumption or hypothesis, and not a judgment.

§ 121. TWOFOLD DIVISION OF THE RULES OF LOGIC.—We propose, therefore, to regard the rules of Logic as legitimately extending to all the processes of ratiocination; and hence as including all rules necessary to direct us in the right use of terms as instruments of ratiocination. They will include, therefore, not only the rules directly governing the process of inference, but also those governing the statement of the premises. The latter—which will first be considered—will be called the "Rules of Judgment," the former, the "Rules of Inference."

§ 122. RULES OF JUDGMENT.—The rules of judgment have for their object, not the forming of right, but the prevention of wrong judgments. Judging is a natural and involuntary operation of the mind. But in the ordinary processes of the mind we are apt to go astray in our judgments; and the object of the rules of judgment is to guard against this infirmity by preventing false judgments, or, where they occur, by detecting them.

§ 123. RULES OF INFERENCE.—The rules of the syllogism given in a previous chapter
THE RULES OF LOGIC

cover all cases of inference except conversion *per accidens*. But these rules are needlessly complex, and may be advantageously replaced by the rules of substitution, which include all inferences whatever, and are simpler both in their expression and application than the old rules, of which they are but another expression. The rules of the syllogism, however, are of such familiar use by logicians, and are so wrought into the terminology and literature of Logic, that a familiar acquaintance with them is essential to the logical student; for whom also it will be necessary to recognize clearly the substantial identity of the two processes.

§ 124. Fallacies of the Syllogism, All Resolvable into Fallacies of Substitution.—This is especially important with reference to the violations of the rules of the syllogism, or, as they are called, the fallacies of the syllogism (§104 et seq.). These are of frequent occurrence, and are familiarly known by technical names; and as these have become firmly established in logical terminology by a use of many centuries, they must, of course, be retained. It will be of advantage to the student, therefore, to have pointed out to him that all these fallacies are simply cases of illicit substitution; which can be readily shown.

Thus, *c. g.*, the fallacy of an ambiguous
middle term (Quarternio Terminorum) consists simply in the substitution of a new term, having the same verbal sign as in the original, but a different meaning—as in the examples given.

The case of undistributed middle—as, e. g., "X is Y, Z is Y .\: Z is X"—consists in the illicit substitution of species for genus in the predicate of an affirmative proposition (i. e., X for Y in the minor premise).

In the case of illicit process of the minor term,—as, e. g., "Y is not X, some Z is Y .\: Z is not X,"—genus is illicitly substituted for species in the subject of an affirmative proposition (i. e., Z for "Some Z" in the minor premise).

In the case of illicit process of the major,—as, e. g., "Y is X, Z is not Y .\: Z is not X,"—genus is illicitly substituted for species in the predicate of a negative proposition (i. e., X for Y in the minor premise).

In the case of negative premise, if the conclusion be affirmative,—as, e. g., "Y is not X, Z is not Y .\: Z is X,"—genus is substituted for species in the predicate of a negative proposition (i. e., Not-X for Y in the minor premise). If the conclusion be negative,—as, e. g., "Y is not X, Z is not Y .\: Z is not X,"—the fallacy will consist in the illicit substitution of one for another of two unrelated terms (i. e., X for Y); and the same will be true of the other cases, if any there be.
§ 125. The Laws of Thought.—The rules of Logic are founded upon what are called the primary Laws of Thought, viz.: (1) the Law of Identity (or rather the Law of Equivalence); (2) the Law of Contradiction; and (3) the Law of Excluded Middle; the first of which governs the process of Inference, the last two, that of the Judgment. The corresponding fallacies consist in their violation.

These laws may be enunciated in a form to make them of practical utility, as follows:

(1) The Law of Identity.

Significates (i.e., things or quasi-things) remain the same though denoted by different terms.

Hence terms denoting the same significates may, to the extent of their equivalence, be used interchangeably, i.e., the one substituted for the other.

The mathematical axiom that "things equal to the same thing are equal to each other" is merely a special application of this principle, its meaning being simply that terms denoting the same class of significates are equivalent to each other.

It is obvious, therefore, that this law is not adequately stated (as is sometimes said) by the equation, $A = A$, but rather by the equation, $A = B$; both terms being supposed to denote the same class of significates, and the term $B$
to be either A, or any other vocable or sign denoting the same significates.

(2) **The Law of Contradiction, or rather the Law of Non-Contradiction.**

A term, and its negative, or contradictory, cannot be predicated universally of any term.

This law and the next are often misstated.

(3) **The Law of Excluded Middle.**

Of two contradictory propositions, one must be true; or symbolically: "Either A is B," or "Some A is not B."  

II

**Rules of Judgment**

§ 126. **Rule I. Terms to be Significant.**

In every logical proposition—by which is meant every proposition to be used in ratiocination—the terms must be significant, i.e., must have definite signification.

This rule follows from the definition of the term and of the proposition; for unless the word or vocable has such definite signification there is no name, and consequently no term or proposition, or valid ratiocination. The violation of this rule may be called the Fallacy of Non-significance or Nonsense.

**Rule II. Terms to be Rightly Defined.**

Terms used in ratiocination must not only have...
a definite signification, but the signification must be legitimate, i. e., they must not be falsely defined. This implies (1) that a term shall not be used in an improper sense, i. e., in a sense not permitted by the usage of the language; and (2) that the term shall be so defined as to signify a real concept; or, at least, that the contrary shall not affirmatively appear.

The violation of this rule will be called the Fallacy of False Definition.

Rule III. Premises not to be Illicitly Assumed.

A proposition that is obviously untrue, or that can, on logical principles, be affirmatively shown to be untrue, cannot be legitimately used as a premise.

The violation of this rule is called the fallacy of "Begging the Question," or Petition Principii; and this and the fallacies resulting from the violation of Rules I. and II. may be classed together under the general head of Illicit Premises.

Rule IV. Premises to Correspond to the Thesis or Issue.

In all ratiocination—if designed to be fruitful

1 The unnecessary use of a term in a sense not justified by usage is commonly indicative either of mental incapacity or fallacious intent; and should therefore be forbidden, as to children we forbid the use of deadly weapons, or to all the possession of counterfeiters' tools.
—the premises, and, consequently, also the conclusion, must correspond to the Thesis or Issue, whether that be expressed or understood, or merely determined by the conditions of the problem.

By the thesis is meant the proposition to be demonstrated; by the issue, the thesis and the anti-thesis, or contradictory, considered together with a view of determining whether the one or the other is true.

With regard to nearly all subjects presented to us for investigation the material question at issue is more or less definitely determined by the conditions of the problem; and hence it is said, "A prudent questioning is a kind of half knowledge" (Prudens interrogatio est dimidium sapientiae). Where the issue is thus determined, it constitutes the real issue, or thesis and anti-thesis of the problem. In other cases it must be determined by agreement, or by actual intention, either expressed or understood. In many cases it is not formally stated, but we ascertain it, for the first time, from the use made of the conclusion.

The fallacy resulting from a violation of this rule—if we assume there is no fallacy in the inference—will necessarily involve a departure from the thesis or issue, both in the premises and in the conclusion. With regard to the premises, it is called the fallacy of Mistaking
the Issue; with regard to the conclusion, that of Irrelevant Conclusion; and in either case, Ignoratio Elenchi.

III

RULES OF INFERENCE

§ 127. All inference, as we have observed, may be resolved into the process of substituting for terms other terms of equivalent ratiocinative value. There is an apparent exception in the case of conversions of propositions, but the exception is only apparent (§ 79). To conform to usage, however, the rule for conversion will be given, though in fact, as explained, the illicit conversion of a proposition is simply a case of illicit substitution of terms.

Rule V. CONVERSIONS TO BE ILLATIVE.

A conversion, to be legitimate, must be illative, i.e., the truth of the converted must be implied in the original proposition.

The violation of this rule may be called the Fallacy of Conversion, or simply Illicit Conversion. It can occur only in the simple conversion of a universal affirmative or a particular negative proposition (e.g., "Y is X," "Some Y is not X"). In the former case the fallacy will consist in the substitution of genus for species (X for Y) in the subject, and of species for genus (Y for X) in the predicate of a universal
affirmative proposition, thus doubly violating the first rule of substitution. In the latter ("Some Y is not X") X is substituted for Y in the subject, and Y for X in the predicate, though neither is necessarily, and one at least cannot be, a species of the other; which is a violation of the next rule.

Rule VI. Equivalence of Terms to be Observed.

In all substitutions the substituted term must be equivalent in signification—i.e., equivalent in ratiocinative value—to the term for which it is substituted.

The violation of this rule by the substitution of a new term is called the Fallacy of Illicit Substitution.

The rule will cover all cases of legitimate substitution of terms whatever; but it is obvious, where an ambiguous term is used in a different sense from that originally adopted, that a new term is in fact illicitly substituted. We must add, therefore, as a corollary the following:

Rule VII. The Sense of Terms to Remain Unaltered.

Every verbal expression, whether a term or proposition, shall, throughout the ratiocination, be used in the sense originally given to it.

The violation of this rule constitutes what is called the Fallacy of Equivocation, which is to
be regarded as a species of Illicit Substitution; and of this there are two kinds: the first consisting in shifting the sense of an ambiguous term, which is called the *Fallacy of Ambiguity*; the second, in shifting the meaning of what is called an *amphibolous* sentence, which is a sentence equivocal by reason of its grammatical construction, as, *e. g.*, the sentence, "'The Duke yet lives that Henry shall depose'"; which may mean either that the Duke shall depose Henry, or Henry the Duke. If construed in the former sense, the subject of the proposition is, "'The Duke that shall depose Henry'"; for which under the latter construction is substituted, "'The Duke that shall be deposed by Henry.'" This is called the *Fallacy of Amphibology*, or, perhaps better, of *Amphiboly*. But these fallacies are of essentially the same nature, and will be classed together under the one head of Equivocation.
PART II
THE DOCTRINE OF FALLACIES

CHAPTER VIII
DEFINITION AND CLASSIFICATION OF FALLACIES

§ 128. Definition of Fallacies.—A fallacy may be defined as a false semblance of valid ratiocination; to which it bears the same relation as hypocrisy, conscious or unconscious, to virtue. Fallacy is therefore a species of error, whose specific difference consists in its semblance of right reasoning and its consequent liability to be mistaken for it.¹ It may

¹ Hobbes, with his usual acuteness, thus clearly explains the distinction between error and fallacy:
“'When we reason with words of general signification (universalibus) and fall upon a general conclusion (conclusionem universalum) which is false, though it be commonly called error, it is indeed an absurdity or senseless speech (oratio insignificans).”—Lev., chap. v. According to this view, all fallacies are absurdities, i. e., they necessarily involve either a contradiction, or the use of non-significant or senseless words.
consist either in a *false judgment* or a *false inference*. But, it will be remembered, the terms *judgment* and *inference* in the logical sense denote, the one *intuitive* judgment, and the other *illative* inference. Hence, when we speak of a *false judgment* or *inference*, we do not mean a real judgment or inference that is untrue (which would involve a contradiction of terms), but—as when we speak of a false prophet—a pretended or simulated judgment or inference that is not really such.

§ 129. Classification of Fallacies.—All fallacies must consist in the violation of some one or more of the rules of Logic, and hence may be correspondingly classified. Such a classification has, indeed, already been substantially effected in our statement of the logical rules; where, under each rule, the corresponding fallacies have been named. It remains, therefore, only to arrange them in convenient order, which is done in the table that follows:

*Table of Fallacies*

§ 130. Fallacies of Judgment.

I. Illicit Premises.

(1) Fallacies in Definition.

Nonsense (or Non-Significance).

False Definition.

(2) Illicit Assumption of Premise (*Petitio Principii*).
II. Mistaking the Issue, or Irrelevant Conclusion (Ignoratio Elenchi).

§ 131. FALLACIES OF INFERENCE, OR ILICIT SUBSTITUTIONS.

I. Illicit Conversions of Propositions.
II. Illicit Substitutions of Terms; Scil.
    (1) Of Vocal Signs, or Vocables.
         Formal.
         Material.
    (2) Of Notions, i.e., of Senses of Terms
         (Equivocation, Homonymia et Amphibolia).

§ 132. OBSERVATIONS ON THE FALLACIES.
—Of the two principal kinds of fallacies contained in the above table, the first—excepting the Fallacy of Irrelevant Conclusion—consist in the illicit assumption of the propositions to be used as the premises of ratiocination. But false or nonsensical propositions do not of themselves constitute fallacies, but only by reason of their use as judgments; for, according to our definition, a fallacy is a false semblance of ratiocination, and therefore cannot exist except as part of ratiocination. Hence we are not concerned with the truth or falsity or the absurdity of any proposition that may be asserted by any one, unless it be used as an independent judgment or as the premise of an
argument, in which case its pretensions may be examined, and, if found to be baseless, it may be challenged as illicit.

Where such an assumed premise is either non-significant or involves a false definition, it is in itself a fallacy, and therefore entitled to an independent rank as such. But such fallacies are innocuous if the sense of the terms be preserved unaltered throughout the ratiocination. For all conclusions in which they are involved must necessarily be without significance, or, in other words, nonsensical, and hence unsusceptible of use. But, as will be seen at large as we proceed, the conclusions from such premises, being in themselves unsusceptible of use, are invariably used as equivalent to other and significant propositions, and thus inevitably result in the Fallacy of Irrelevant Conclusion, or Ignoratio Elenchi, which consists in substituting for the conclusion another proposition (i.e., the true thesis); and which, though for convenience treated separately, may itself always be resolved into the Fallacy of Illicit Substitution, i.e., into an illicit conversion, or an illicit substitution of a term. And the same observation is true generally, though not universally, of illicit assumptions of false premises. These, if regarded as mere hypotheses, and if no misuse be made of the conclusion, are not illegitimate; but, it will
be seen, a conclusion deduced from such premises almost invariably either comes in conflict with some inconsistent fact, or otherwise fails to be sufficient for the purposes the reasoner has in view; and thus, almost inevitably, it is treated as equivalent to some other proposition, thus again presenting a case of *Ignoratio Elenchi*. Hence—if, as we conveniently may, we regard all assumed propositions as mere hypotheses, and therefore as not illegitimate, unless an ill use be made of the conclusion—all illicit assumptions of premises must necessarily result in an *Ignoratio Elenchi*; which, as we have observed, must necessarily consist either in an illicit conversion or the illicit substitution of a term. Hence, as all valid ratiocination consists in the substitution of equivalent (§ 78), so all fallacy must consist in the substitution of non-equivalent terms.

Hence the simplest and most scientific classification of fallacies would be to regard them all as species of *illicit substitution*—that is to say, as cases, either of *illicit conversion* of propositions or *illicit substitution* of terms; and that we have adopted a different mode of classification is due simply to the consideration that we may thus more conveniently exhibit the different sources of fallacy. Hence, as we proceed, it will be found that the several fallacies all have a tendency, as it were, to run
into each other; which mainly results from the fact that they are all in their essential nature the same, differing only in the peculiar sources in which they originate; though partly also from the fact that, in general, fallacious arguments are not explicit, and the fallacy may vary according to the manner in which we may express them.

In our classification of the fallacies we have distinguished as a class the fallacy of "Mistaking the Issue, or Irrelevant Conclusion," thus apparently including two separate fallacies. But this is only apparently so. For unless there be some fault in the inference—which would constitute another kind of fallacy—the conclusion and the premises must necessarily correspond, and we may therefore regard either the illicit assumption or the illicit conclusion as constituting the fallacy. If we regard the latter as the fallacy, it necessarily resolves itself into a case of illicit substitution. But, for convenience, we regard it as relating to the premises, and thus regarded, it consists in the illicit assumption of one proposition in place of another—i.e., of the actual premise for some other proposition more or less resembling it which is admitted.

In concluding these introductory observations I would refer the student to what is said in the conclusion of the Introduction, and
which, for convenience of reference, is here repeated:

"In our treatment of the subject, the several fallacies will be illustrated almost exclusively by examples taken from current theories of Politics and Morality. Our examples will therefore consist, not of mere trivialities, such as are so commonly used in works on Logic, but of fallacies that, in perverting moral and political theory and in corrupting practice, have dominated, and still continue to dominate, the fortunes of the world. They come to us, therefore, as veterans in the army of what Hobbes calls the 'Kingdom of Darkness,' crowned with the laurels of victory" (§ 13).

Among these theories there are two fruitful, above all others, in examples of logical fallacy—namely, the modern doctrine of Absolute Sovereignty, and the Utilitarian Theory of Morality; the former of which may be expressed in the proposition that "Sovereignty is, in its essential nature, an absolute power, and, as such, unsusceptible either of limitation or division"; the latter, in the proposition that "General Utility is the true and only standard of justice and injustice, and of right and wrong generally." Most of our examples will be taken from these theories; and these, and other current theories used for the same purpose, will be found not only to serve as the
most effective means of illustrating the nature of the several fallacies involved, but also to enable us to perceive the frequent use and formidable influence of fallacy upon political and moral speculation, and to realize how disastrously and commonly the most vital affairs of mankind are thus affected.
CHAPTER IX

NON-SIGNIFICANCE, OR NONSENSE—FALLACY OF

§ 133. The nature of this fallacy is explained under Rule I. of the Rules of Logic. The fallacy is of two kinds; namely, (1) where a term is used that has an impossible or absurd meaning or no meaning at all—which constitutes the Fallacy of Nonsense in the narrower sense of the term; and (2) where an ambiguous term is used without definition—which is called the Fallacy of Confusion. But, logically, the two kinds are of essentially the same nature, and hence are classed together under the general head of Non-significance or Nonsense. For the purpose of illustrating their nature, they will, however, be considered separately.

1. The Fallacy of Nonsense

§ 134. In dealing with concrete matters, it is difficult to use nonsensical speech without

1 According to Hobbes (cited supra, § 128, n.), all fallacies, in their ultimate analysis, may be reduced to this head.
discovering it; and hence the kind of nonsense to which the term is colloquially applied is generally of an obvious and transparent character. But when we come to deal with abstract terms, or terms of second intention, such as are constantly used in Morality, Politics, and Metaphysics, the case is quite different. For here not only are we liable constantly to use nonsensical or non-significant terms, but it often requires the most searching and difficult analysis to discover that we have done so. Hence, the nonsense of which we are to discourse is something very different from the nonsense of colloquial speech; which is generally so obvious that only foolish people can fall into it, or, at least, persist in it. It is a kind of nonsense that constantly imposes itself upon the most eminent statesmen, jurists and philosophers, and even upon the most acute logicians. To escape it altogether a man must be endowed with more than mortal sagacity, and hence the fallacy may be illustrated by examples from the writings of the most eminent men.

Examples

§ 135. SOVEREIGNTY.—The most striking example of this fallacy is presented by the modern doctrine of Absolute Sovereignty (§ 132), a doctrine almost universally received by modern political writers, and which (with an
exception, to be touched upon under the next head) has contributed more than any other cause to the corruption of political philosophy and practice. This will require some explanation.

The term Sovereign, in its original and proper sense, denoted merely a single ruler or monarch, and Sovereignty, the power of this monarch. But in modern times the application of these terms has been much extended, and the latter term is now used in many different ways; of which four may be distinguished, namely: (1) *Personal Sovereignty*, or the power of an absolute monarch—otherwise known as "the Divine Right of Kings"; (2) *Corporate Sovereignty*, or the Sovereignty of the government, whether monarchic, aristocratic, democratic, or mixed; (3) *Popular Sovereignty*, or the Sovereignty of the state or people; and (4) *The Sovereignty of Right or the Law.*¹ To which may be added as many other senses as abstractions can be imagined for the purpose—as, *e.g.*, the Sovereignty of Reason, or, in a theocracy, the Sovereignty of God. All these different

¹ This expression originated with Aristotle: "Moreover, he who bids the law to be supreme, makes God supreme; but he who trusts man with supreme power gives it to a wild beast, for such his appetites often make him; passion, too, influences those who are in power, even the very best of men; for which reason the law is intellect free from passion."—*Politics*, iii., xvi.
senses of the term are inconsistent with each other; and all except the first (now happily obsolete) are—in their direct sense—without definite signification, or, in other words, nonsensical. For the government or state, and likewise right or law and reason, are purely imaginary or fictitious persons, existing only in contemplation of mind—i. e., they are quasi-persons only; and the power of such fictitious or imaginary beings must be as imaginary as themselves. For the government or state or a corporation cannot, properly speaking, be said to have rights, or will, or power, or conscience, or other human attribute; and when, otherwise than as a mere figure of speech, we speak of such quasi-persons as having such attributes, we talk pure nonsense. And so with reference to the sovereignty of God, though the same observation is not literally true, yet practically, as we can know but little of His will, or the exertions of His power, the term, as generally used, carries with it no meaning.

The following examples are in effect identical with the above:

(1) The doctrine of Kant, Rousseau, and others, that the will of the government or the state is to be regarded as "the united will of the people"; which is obviously a mere fiction, and, construed literally, not only false, but impossible.
(2) The proposition of Hobbes, that the effect of the institution of government was to create not merely "a consent or concord" of the people, but "a real unity of them all in one and the same person."

(3) The equivalent proposition of Bluntschli and others, that the state is an "organized being" or "organism," having a soul and a body, a conscience and active powers, and also a will different from the wills of the individuals composing it.

(4) And finally the celebrated theory of the Social Compact or Contract, which served Hobbes, Locke, Rousseau, and others as the foundation of their respective reasonings; and from which, as a premise, their several essentially different and antagonistic theories are, with equal felicity, deduced.

§ 136. Of Legal Fictions.—These are all examples of what lawyers call legal fictions; which are at least as common with the philosophers as with the lawyers. In all of them—except the last—the government or state is regarded as a body politic or corporation; which

1 The difference between the lawyers and the philosophers in this respect is that by the former the fiction is always recognized as such, and used merely as a convenient mnemotechnic device. It is also used, not as a universal, but as a particular proposition—its use being restricted by the maxim, "In fictione juris semper æquitas." But the use of it by philosophers is often the reverse.
is defined as a fictitious or imaginary person, existing only in contemplation of mind,—i.e., as a *quasi*-person,—and the definition is, in fact, but a bold metaphor. Hence, as we have said, the power of this fictitious or imaginary being is as imaginary as itself. For human power can exist only in actual human beings; and though for convenience we may speak of the power of the government as of that of any other corporation, yet the expression is always to be understood as really denoting the concurrent powers of certain individuals in the government. Hence, when we attribute to the state or government, or any other corporation or fictitious entity, *will, conscience, soul, body, sex*, or other human faculty, feeling, or quality, we speak figuratively, and, as in all cases of figurative language, if literally, absurdly. The examples cited may therefore be more specifically assigned to the class of fallacies called by the old logicians the *Fallacy of Figure of Speech* (*Fallacia Figuræ Dictionis*), *(infra, § 203)*.

With regard to the doctrine of a social compact, it has not the excuse of being even figuratively true. Like the fiction of the English law that husband and wife are one, it is simply an undisguised, recognized absurdity, assumed as a first principle. That it should ever be asserted would, were it not for experience to the contrary, be simply incredible.
§ 137. THE DARTMOUTH COLLEGE CASE.—A similar example of this fallacy is presented by the decision of Chief Justice Marshall in the Dartmouth College case (4 Wheat., 518), where it was held that an act of the Legislature re-organizing a collegiate corporation was in conflict with the provision of the Constitution of the United States forbidding enactment by a State of any law "impairing the obligation of contracts." It was not perceived that a corporation, being a fictitious person, is not capable of having any rights, except as representing real persons, and that its so-called rights are in fact merely the rights of its stockholders or other parties interested in it. But in eleemosynary corporations there are no private parties interested, and hence the supposed rights of the corporation are in fact those of the State, and consequently subject to its disposition. For it is absurd to speak of rights that have no real owners; and to such rights the Constitution—which was designed to protect the rights of real persons—can have no application. The decision was therefore simply a case of the Fallacy of Nonsense, of the kind called F. Figure Dictionis.

§ 138. OBSERVATIONS ON THE FALLACY OF NONSENSE.—It may be observed here by the reader, who is somewhat familiar with Logical Doctrine, that the Fallacy of Nonsense is ap-
parently a new kind of fallacy, not to be found in the books; but this is very readily explained. For, as we have observed, a conclusion involving a nonsensical term, being itself nonsensical, can in its proper sense, or rather nonsense, be of no use for any purpose, and hence is always used as equivalent to some significant proposition, and thus becomes an *Ignoratio Elenchi*. Thus the doctrine of Absolute Sovereignty, like other nonsensical theories, is in itself innocuous, and becomes otherwise only by illicit use. There can be no harm in saying that Leviathan, the creature of our imagination, is vested with unlimited power, or even to say with Hobbes that he is a "mortal god," and therefore omnipotent. For his power, if left to himself, is no more formidable than that of the wooden or brazen gods of the heathen. But as in the latter case the power of the god is, in practice, the power of the priest, so the imaginary power of Leviathan is but a word used to cover the actual power of some officer or officers of the government; and to them the meaning of the doctrine is: "You must not resist us." Hence, invariably, a nonsensical term is used only in the argument, and the conclusion is always used as equivalent to some other and significant proposition, thus making a case of Irrelevant Conclusion, or *Ignoratio Elenchi*; under which head it is
commonly treated. Of this numerous examples will be given in the sequel.

2. The Fallacy of Confusion

§ 139. This fallacy is recognized in the books as one of the most common and pernicious; and, indeed, it is a commonplace in philosophy that the use of undefined terms is one of the most fruitful sources of error. The nature of the fallacy is explained under Rule I. of the Rules of Logic. A few examples will be sufficient to illustrate its nature.

Examples

§ 140. Utilitarianism.—The most serious example of this fallacy is presented by the theory of Utilitarianism (§ 132 ad fin.), which for the greater part of a century has exercised a predominating and pernicious influence over English thought. The theory, briefly stated, is that general utility is the paramount and sole standard of right and wrong and of the just and unjust. But the term "general utility" has no definite meaning; because it is impossible to determine from it who are the people whose utility or welfare is to be considered—whether a mere majority or less, or two thirds, or three fourths, or other proportion; and
hence the proposition must be regarded as non-significant or nonsensical.

§ 141. EDUCATION.—So he who asserts the benefit of education is, in general, talking nonsense. For education is but the development of character,—mental, moral, and physical,—and may be either good or bad. For there is an education of the thief, of the bully, of the tramp, as well as of the honest man, of the hero, of the efficient man, or of the scholar, or statesman, or philosopher. And so, even among legitimate kinds of education, there is an education of the mechanic, of the farmer, of the laborer, of the lawyer, of the doctor, and many other kinds. Consequently, when one asserts the benefit of education generally, without defining the term, the proposition is nonsensical.

§ 142. PROTECTION.—So the man that asserts that he is in favor of the protection of American industries is, in general, talking pure nonsense. For there are many kinds of protection, as, e. g., (1) The prohibition of all foreign imports that compete with our own industries; (2) the equalization of the cost of production; and (3) the encouragement of infant industries; and until we are told which of these various kinds of protection is intended the proposition conveys no definite meaning.

§ 143. EXPANSION.—So when an American
announces himself as an advocate of territorial expansion he is, generally, talking nonsense; for there are many kinds of expansion, among which three may be especially distinguished, namely: (1) The acquisition of contiguous homogeneous territory essential to the safety of the government, as, e. g., in the case of the purchase of Louisiana; (2) the acquisition of contiguous and homogeneous territory desirable as giving room for the expansion of population, but not essential to the safety of the government, as, e. g., the acquisition of California, New Mexico, etc.; and (3) the acquisition of territory far removed from our own, of a climate unsuited to our people, and inhabited by an alien and non-assimilable race. Such a country must be governed by despotic power, and its acquisition must therefore be distinguished from other kinds of expansion by the name of Imperialism.
CHAPTER X

FALLACY OF FALSE DEFINITION

§ 144. The nature of this fallacy is explained under Rule II. of the Rules of Logic. As there explained, the fallacy is of two kinds—consisting, the one in the use of a term in an improper sense, *i. e.*, in a sense not permitted by the usage of the language—the other, in using a term in an unreal sense, *i. e.*, as denoting a notion to which there is no corresponding reality.

The former kind of the fallacy is not admitted by logicians generally; for it is an unfortunate delusion of philosophers that they are at liberty to define a term as they please. But whether this claim be admitted or otherwise, it has been the source of infinite error; so that the violation of the rule, if not regarded as a fallacy, must at least be regarded as a most prolific mother of fallacy. For where a term is used in a novel sense, though clearly defined, it is hardly within the power of the human intellect
to emancipate itself from the influence of its usual and proper signification. Hence, inevitably, the use of improper terms will result in the fallacy of Ignoratio Elenchi.

Examples

§ 145. Whately’s Definition of Logic. —Whately’s definition of Logic as “the science and art of reasoning,” and of Reasoning as consisting solely in syllogistic inference, presents an instructive example of the Fallacy of False Definition. This definition excludes from the province of Logic the doctrine of Judgment, and, as involved in this, the doctrine of the Term, and also that of the fallacies called Non-logical or Material, thus mutilating it of its most vital parts. But these subjects are invariably treated of by the logicians, including himself, and—as is now generally admitted—belong to logical doctrine; which is an effective reductio ad absurdum of the definition.

§ 146. Stewart’s Definition of Reasoning.—From the same false definition of Logic, and of reasoning, Dugald Stewart deduces the paradoxical conclusion that not only Logic, but reasoning itself, is but of little utility; which constitutes a still more effective reductio ad absurdum of the falseness of the definition.¹

¹ "Of the different elements which enter into the composition of reason, in the most enlarged acceptation of the word,
§ 147. Locke’s Attacks on Logic.—Locke’s diatribes against Logic had their source in the same false definition of Logic as being merely the doctrine of syllogism. But, strangely enough, at the end of his work he gives a correct definition of it; which, as we have seen, he takes for an invention of his own (§ 110).

§ 148. Mill’s Definition of Logic.—According to Mill’s definition, “Logic is not the science of belief, but is the science of proof or evidence,” or, as otherwise expressed, “the science of the operations of the understanding which are subservient to the estimation of evidence.” But bearing in mind the essential difference between judgments and assumptions it will be observed—if we leave out of view axioms, which are to be regarded merely as laws or conditions, to which the mind operating intelligently must conform—that the former constitute the first principles of all demonstrative or apodictic reasoning, and therefore necessarily fall within the province of Logic; but, with regard to assumptions, that Logic is not concerned with the evidence of their truth. But the term, evidence, in its proper sense, relates exclusively to assumptions or propositions the power of carrying on long processes of reasoning or deduction is, in point of importance, one of the least.”—Phil, of the Mind, v. ii, p. 154.
in which the significative relations of the term are not intuitively perceived; and hence, with regard to such propositions, the respective provinces of Logic and of the other sciences are clearly defined. The latter deal with the evidence of the propositions assumed; the former, exclusively with inferences from them, upon the assumption or hypothesis that they are true. Hence the definition of Mill precisely reverses the several functions of Logic and of the other sciences that furnish it with assumed propositions as premises.

§ 149. Hamilton’s Definition of Logic. —The definition of Logic as “the science of the laws or forms of thought” may be cited as another example. Logic is concerned, not with all thought, but with a particular kind of thought only — namely, reasoning; and it is concerned, not only with the forms, but with the matter of reasoning. The definition is therefore at once too wide and too narrow; it would include, e. g., rhetoric and grammar, and would exclude the best part of Logic.

§ 150. Definition of the Law. —A most striking example of the Fallacy of False Definition is presented by the definition of the Law, invented by Blackstone and adopted as the first principle of jurisprudence by Bentham and Austin. According to this definition, the law is merely an expression of the will of the
government—an obviously false and illegitimate definition. Yet the theory of Bentham and Austin, based on this definition, has absolutely dominated jurisprudence in England and this country for nearly a century; and, as the result, English and American jurists and publicists have lost mental touch with the jurists of other countries and ages; and have thus, with reference to scientific jurisprudence, been rendered incapable of dealing with this great and important subject. And indeed the effect of the theory on the practical administration of justice has been scarcely less deleterious.

§ 151. The Theory of Private Utility. —Another conspicuous example of this fallacy is furnished by the theory of individual utility assumed by Hobbes, Bentham, and Austin as the first principle of Morality and Politics; in which self-interest is regarded as the sole possible motive of human conduct, and right and wrong, just and unjust, and good and evil are defined as consisting in conformity or nonconformity to that interest.

§ 152. The Greatest Good of the Greatest Number.—Bentham also inconsistently held the theory that "the greatest good of the greatest number" is the true standard of Morality; which must either be regarded, like the theory of General Utility, as simply nonsensical, or as holding that the standard of
right and wrong and of the just and unjust is the good of the majority. An execrable doctrine; for it cannot be asserted that the life or faculties or property of an innocent man can be converted to the use of another or of others, except in the case of a clearly defined right in the one and an obligation to submit to it in the other.

§ 153. MAINE’S DEFINITION OF THE LAW OF NATURE.—Another example is presented by the peculiar and curious view taken by Sir Henry Maine of the term *Jus Naturale* as used by the Roman lawyers, and its equivalent, the Law of Nature, or Natural Law, as used by modern jurists and philosophers. This notion, he erroneously assumes, had its origin in the supposed *state of nature*; which doctrine, he says, the Roman jurisconsults borrowed from the Greek philosophers. But the term *Jus Naturale*, or Law of Nature, is one of the comparatively small class of terms whose meaning is perfectly definite and settled. As used by jurists, it is but another name for Natural Justice,¹ or Right Reason applied to the jural

¹ Hobbes’s *Lev.*, chap. xxvi. “It is not used among them that be learned in the laws of England to reason what thing is commanded or prohibited by the law of nature.” But, “when anything is grounded on the law of nature, they say that reason will that such a thing be done; and if it be prohibited by the law of nature, they say it is against reason” (*Doctor and Student*, chap. v.). “True law is right reason
relations of men; which, as universally held by them, "is part of the law of every common-wealth in the world."

conformable to nature" (Cicero, De Rep.). "Right reason is what we call law" (id., De Leg.). "Natural law is the rule and dictate of right reason" (Taylor, Elements of Civil Law). "The law is intellect free from passion" (Arist., supra, § 135 n.).
CHAPTER XI

ILlicit Assumption of Premises (Petitio Principii)

1. Of the Nature and Several Forms of this Fallacy

§ 154. This fallacy may occur in various ways, and it would therefore be an endless task to enumerate or classify all its different forms; nor would there be any advantage in doing so. There are, however, several forms of the fallacy that, on account of their frequent occurrence and their powerful influence over the minds of men, demand a particular consideration, and to these our attention will be directed.

§ 155 (1). Illicit Generalization.—The most important of these, which may be called the Fallacy of Illicit Generalization, consists in the use of a universal proposition in cases where the corresponding particular proposition is alone admissible. This fallacy is one of the most common and formidable, not only in popular discourses, but in more pretentious
works on Politics and Morality; for almost all the wisdom of common sense is embodied in this sort of propositions, *i. e.*, *particular* propositions assumed to be *universal*. Such propositions may, indeed, be used with profit by men of sense in practical affairs; as, in general, when a question presents itself it is easy to perceive whether the principle should be applied or not; or, if a mistake be made, it is corrected by experience; but the masses of men are easily misled by them. Hence they serve well for rhetorical purposes; for the hearer, unless of a critical mind, will in general accept them without hesitation.

**Examples**

§ 156. **Commonplaces.**—The most important cases of this fallacy occur in the use of **Commonplaces**; by which is meant, opinions current among men generally, or particular classes of men, and used as premises for reasoning.¹ These are commonly founded upon some truth which they purport to express, and to which they more or less nearly approximate;

¹ Hence Bacon, as a useful rhetorical device, recommends the preparation of tables of Commonplaces, of which he gives an example in his *De Augmentis*; wherein should be arranged, for the use of speakers and writers, in parallel columns, arguments *pro* and *con*, or *theses* and *anti-theses*, on all questions of general interest,
so that there is here, as "in all things evil, a soul of truth." But they are hardly ever universally true; and therefore to assume them as universals is illicit.

§ 157. Popular Proverbs.—Of these commonplace, the most striking examples are furnished by popular proverbs; and of these, as illustrating precisely the nature of such maxims, two may be cited that, in their literal expression, are contradictory, but, as maxims go, may both be said to be true, i.e., they are each true in certain cases, but neither universally. They are the old adages, "Never put off till to-morrow what you can as well do today" and "Never do to-day what you can as well put off till to-morrow"; the first of which points out the danger of procrastination, the latter, the danger of committing ourselves before necessity requires. It may be readily seen that, according to circumstances, either of these may serve as a useful hint for conduct; but, in using it, the caution of the nautical philosopher is to be observed, that "the bearing of the observation lies in the application of it."

§ 158. Legal Maxims.—Another striking illustration of the same class of propositions is furnished by what are called the maxims of the law; which, in general, are true only as particular propositions, i.e., only in particular cases,
but are habitually spoken of by legal writers as "first principles," analogous to the maxims of science; though every competent lawyer is familiar with the fact that they admit of numerous exceptions. A very large proportion of the so-called principles of the law, and of the rules founded upon them, are of precisely this nature, *i. e.*, admit of exceptions, and are, therefore, true only as particular propositions. And it is also a fact that many of these principles and rules are opposed by others, equally approved, that are contradictory to them. Hence, if we regard bulk only, the greater part of the law might be readily and advantageously arranged in a table of contradictory commonplaces,—*i. e.*, a collection of theses and anti-theses,—as suggested by Bacon in the *De Augmentis*: wherein, under each topic, one column should represent the one side and the other, the other, of the various questions that may arise in litigation. The cases might also be arranged in the same way.

The above examples are all cases of illicit generalization, and will serve to show how widespread is the use of this particular form of illicit assumption of premise. And, it may be added, such is the lack of critical acumen in the generality of mankind, that the fallacy is seldom detected, and consequently it constitutes the most powerful of rhetorical devices.
§ 159 (2). Of the Fallacy of Non Causa pro Causa.—Another form of the Fallacy of Illicit Assumption of Premise is presented by the fallacy called "Non causa pro causa"; which is also called the fallacy of "Post hoc ergo propter hoc." It consists in the illicit assumption that an event preceding another event is the cause of the latter, as, e. g., that a change in the moon is the cause of a change in the weather; or that the fact of thirteen dining together is the cause of any accident that may happen to any one of them; or that the Dog Star is the cause of heat. This is, indeed, one of the most familiar of fallacies in political arguments, where it is common to argue that the condition of the country, whether good or bad, is caused by some particular policy, as, e. g., where it is argued alternately, according to vicissitudes of events, by the one party that a prosperous, by the other that a depressed, condition of affairs is caused by the tariff or other political measure.¹

¹ It will be observed that there are some differences of opinion among logicians as to this fallacy. A distinction is made between what is called the causa essendi and the causa cognoscendi; or between the cause of an event and the cause of our knowing it. These may coincide, as, e. g., when from the fact of its raining in the night we infer that the ground will be wet in the morning; where the rain is both the causa essendi and the causa cognoscendi. But, when, from finding the ground wet in the morning, we infer that it rained during
§ 160 (3). Arguing in a Circle.—Another common form of the Fallacy of Illicit Assumption is presented by the fallacy called arguing in a circle; which consists in assuming for a premise the very proposition to be proved, or one obviously equivalent to it, or one that is formally involved in it.¹ When the argument does not extend beyond a single syllogism it is called a Hysteron Proteron (the First-last).²

§ 161 (4). Question-Begging Terms.—Another very common and very dangerous the night, the *causa cognoscendi* is the wet ground, from which we infer the *causa essendi*, i. e., the rain. Logic is, however, concerned with the *causa essendi* only so far as it constitutes the *causa cognoscendi*; and hence logically the distinction may be regarded as immaterial.

¹ This occurs most frequently in the use of synonyms, and, as observed by Whately, is peculiarly favored by the composite character of our language. It can occur only where the proposition assumed is so obviously equivalent to the conclusion as to be evidently the result of a trick or inadvertence. In general the premises assumed are equivalent to, or imply, the conclusion; and the conclusion is arrived at by the substitution of an equivalent term; which is the very essence of ratio-cination. Such assumptions are not only admissible, but inevitable. Otherwise all syllogisms would be fallacious,—as involving a *petitio principii*; and inference, inconceivable.

² The following is a striking example of this fallacy: "Since every unjust act is inexpedient, then no unjust act is expedient; then no expedient act is unjust; then every expedient act is just." This has been given as a valid argument. But the premise is obviously but an inference from the conclusion, which is the *principle* of the reasoning; and for it the *thesis* has been illicitly substituted as the premise.
form of this fallacy is that of using question-begging terms (which is also a case of the Fallacy of False Definition). It consists either in including in the formal definition of a term some unproved assumption, as being of the essence of the conception denoted, or in using the term without formal definition, as though such assumption were included in its meaning. By this method, the propositions from which our conclusions are to be deduced, instead of being proved as they ought to be, are unconsciously imbibed by the mind, with the definition, or with our conception of the term, and the conclusion thus in effect assumed. The power of this method of persuasion is well understood by many, and unscrupulously used—as, for example, by Hobbes and other supporters of governmental absolutism; who realize the truth of Rousseau’s observation that “the strongest is not strong enough to continue always master, unless he transforms his power into a right, and obedience into a duty.” But with the mass of writers the fallacious process, though none the less efficacious, is entirely unconscious. A notable example of this fallacy is usually given by political writers in their definitions of “the State”; which is simply “an independent society of men,” but is usually defined so as to include in its essence absolute power, or some other theory of the
Any recent work on Politics will serve to illustrate the fallacy.

2. Of the Tests of Illicit Assumption

§ 162. Enumeration of the Tests.—There are numerous tests by which the legitimacy of assumed premises may be determined, of which the most important and familiar are: (1) the "Instance," or "Extreme Case"; (2) the "Burden of Proof," or Onus Probandi; and (3) the Reductio ad Absurdum. These will next be considered.

§ 163. The Instance, or Extreme Case.—This test applies most appropriately to the Fallacy of Illicit Generalization, and is most efficacious in its operation; though, as is observed by De Morgan, it is commonly regarded as not only inadmissible, but impertinent. It consists simply in adducing an exception to the proposition assumed. The subject is admirably treated by the author cited.¹

§ 164. The Onus Probandi.—An extremely effective means of testing the truth of

¹The term "instance" is commonly used as synonymous with "example," but it is said by De Morgan that by the mediæval logicians it was always used to denote an inconsistent example, or, in other words, to denote what we would call an instance to the contrary,—an expression that would have been regarded by them as tautological.

²"The application of the extreme case is very often the only test by which an ambiguous assumption can be dealt
a proposition, and of thus exposing an Illicit Assumption, is often afforded by considering what is the presumption in the case; or, contrariwise, on which side of the question lies the burden of proof, or onus probandi. In general, this is on the party affirming the proposition, and, in the absence of other presumptions, we are always entitled to demand his proofs. This simple test will be sufficient to dispose of all propositions for which proofs cannot be found, but which have been inadvertently assumed; and this test we should always apply to our own reasoning, remembering that "Slowness of belief and distrust are the very sinews of wisdom." But in certain cases, and especially in Moral and Political Science, the test will often have a conclusive efficacy. For in Morality, Public and Private, or in Jurisprudence or Right, the questions presented are generally questions, not of fact, but of right and wrong; and among these there are certain fundamental principles, as, e.g., touching the right of personal liberty or security or self-ownership, with reference to which the presumption is clearly defined, and its contradictory obviously absurd. Of this kind is the general presumption in favor of liberty; which, of with; no wonder that the assumer should dread and protest against a process which is as powerful as the sign of the cross was once believed to be against evil spirits."—Formal Logic.
itself, is sufficient to dispose of numerous and important political theories that, from a neglect to consider the onus probandi, have been carelessly or dishonestly assumed.

§ 165. Of the Reductio ad Absurdum.—This consists in reasoning from the conclusion deduced from the premises assumed to some absurd, or admittedly untrue, conclusion; and this method of refutation will apply not only to the fallacy of illicit generalization, but to all forms of pettio principii whatever. It is, indeed, one of the most efficacious means that Logic has at its command for the detection of fallacy, and will therefore repay an attentive consideration.

Strictly speaking, the phrase would seem to indicate that it applies only to the establishment of the contradictory of the proposition under consideration; but the method has, in fact, a much wider application, and the term, in common use, a corresponding extension. For it is the essential characteristic of all true

1 In the narrower sense, the term reductio ad absurdum is equivalent to the reductio ad impossibile; of which examples are given supra (§ 96, n.). But more generally it is used as including all cases where, from the conclusion of an argument, the contradictory of some admitted proposition—or, in other words, a conclusion contrary to the hypothesis—can be deduced. Hence it is called by Aristotle the "Argument from Hypothesis." (Mansel's Aldrich, App., note I, p. 228.)
propositions that they will be consistent with each other; and it is an almost equally universal characteristic of untrue propositions that they will be inconsistent with other propositions known to be true.

This is particularly the case in all the different branches of the Science of Human Nature; all of whose parts and particular principles are so connected by numerous relations that it is almost impossible to assert an untrue principle without coming in conflict with others that are self-evident, or readily demonstrable, and which have thus come to be universally admitted. Hence it may be said that in Morality or Politics we may set out from almost any principles, provided we hold them with indifference and are capable of abandoning them when shown to be inconsistent with settled principles and known facts. From which it may be inferred that the *reductio ad absurdum* in fact constitutes not only an efficient, but almost an all-sufficient, instrument for the detection of fallacy in Moral and Political Science.

General Examples

§ 166. Locke’s Theory of Simple Ideas. —A most instructive example of Illicit Assumption of Premise occurs in the fundamental assumption of Locke’s theory of knowledge;
which is, that the original notions received in the mind from sensible objects are notions of the qualities of substances, such as color, hardness, etc., which he calls simple ideas; and out of which, he holds, all our notions are compounded. But on reflection it will be perceived that the original or primordial notions of the mind are the composite notions of substances or things; and what Locke calls "simple notions" are the result of subsequent analysis.

§ 167. The Obligation of Contracts.—It is one of the so-called maxims of the law that contracts are obligatory and ought to be enforced (Pacta quaelibet servanda sunt); and this is commonly assumed as a universal proposition, as, e. g., by Bentham and Spencer in the examples given below (§§ 180, 181). But there are innumerable cases in which it is obviously not right that contracts should be enforced, and in which, in fact, the law does not enforce them; which is an effectual refutation of the principle. The true principle is that in case of breach of contract the injured party is entitled to compensation—as in the case of torts—for the detriment suffered by him by the acts of the wrongdoer (i. e., by the making of the contract and its breach).

§ 168. False Assumption of Fact.—This includes innumerable cases, which it would be impossible to classify. One of the most in-
interesting is furnished by Tacitus in his account of the mutiny of the Pannonian legions on the accession of Tiberius,—in the address of the soldier, Vibulenus, to the general, Blöesus. His brother, he said, coming as a delegate from the German army, had been butchered by the commands of Blöesus. "Answer, Blöesus," he said; "where hast thou thrown away his corpse?" By which, says Tacitus, "he raised such a spirit of frenzy and vengeance that had it not been quickly manifested that there was no corpse to be found . . . and that Vibulenus never had any brother, they had gone nigh to sacrifice the general." The example, so far as Vibulenus is concerned, was simply a lie, but, in the soldiers, a fallacy that would have been readily refuted by applying the test of the onus probandi.
CHAPTER XII

MISTAKING THE ISSUE, AND IRRELEVANT CONCLUSION (IGNORATIO ELENCHI)

§ 169. The nature of this fallacy, which is explained under Rule IV. of the Rules of Logic, is precisely expressed by the first of the names we have given it, which is a technical term taken from the law. This differs from the equally appropriate term Irrelevant Conclusion only in this, that the former has regard to the origin, the latter to the outcome of the fallacy. Or, in other words, when we regard the beginning of the fallacy, we call it Mistaking the Issue; when the end, Irrelevant Conclusion; and, in either case, Ignoratio Elenchi. The two names, i.e., Mistaking the Issue and Irrelevant Conclusion, present, therefore, two different aspects of the same fallacy, under each of which it will be convenient to consider it.

§ 170. MISTAKING THE ISSUE.—This, as is well appreciated by the lawyers, is one of the most formidable and most common of all fal-
MISTAKING THE ISSUE

lacies. For the most fruitful of all sources of fallacy is bias or logical dishonesty, of which the expedient of mistaking or misstating the question at issue is one of the most obvious and most potent instrumentalities. And as logical honesty is, in fact, one of the rarest of intellectual virtues, it can be readily understood that the fallacy must be common.

§ 171. FALLACY OF SEVERAL QUESTIONS OR ISSUES.—One form of this fallacy may be identified with the technical Fallacia plurium interrogationum (§ 197), which consists in mixing in one several questions or issues. As defined by Aristotle, it results “from making two questions one, when it escapes notice that there are many, and one answer is given, as if there was one question only.”

The following examples are taken from a recent work:

“‘Did you steal anything when you broke into my house last night?’ ‘Are you the only rogue in your family?’ ‘Have you quit drinking?’ ‘Have you cast your horns?’ (Hence sometimes called Cornutus.)”—(Davis, Theory of Thought, 294.)

The fallacy is readily solved by separating the compound question into its several components,—as, e. g., in the following: Menedemus, Alexino rogante, Numquid, patrem verberare desiisset? inquit, Ncc verberavi, ncc desti; or,
as in the answers of the two thieves to the question: "Did you steal the sheep you have in your possession?"; to which the one answered, "He did n't steal the sheep"; the other, that "He did n't have it."

§ 172. It is added by the author, "All this seems quite frivolous." And another, generally accurate, logician says: "The so-called 'Fallacia plurium interrogationum' has not been noticed in the text, because it is a rhetorical artifice rather than a logical fallacy." (Fowler, Deductive Logic, 150.) But it cannot be doubted that the fallacy, as described by Aristotle, consists simply in mixing several questions or issues in one, and therefore comes under the head of mistaking the issue; or that it is at once a very common and a very formidable fallacy. And especially, it is to be observed, it is the hard fortune of the citizen, in all ages and countries, that, in general, whether by accident or design, no question in practical politics is presented to him that does not involve this fallacy.

Thus, in American politics, for some time after the war, several questions (plures interrogationes) were presented at each federal election, namely: (1) as to the expediency of the protective policy; (2) as to that of the reconstruction policy; (3) as to that of the contraction of the currency; and thus practically
the questions presented to each voter were:
"Are you in favor of all these policies?" or
"Are you against them all?" So in the last
election, the issues presented were equally
numerous—namely: (1) as to the policy of
protection; (2) as to the relative advantages
of the single gold or a bimetallic standard; and
(3)—assuming the desirability of bimetallism
—as to the practicability of adopting it in this
country alone, without the concurrence of
other nations.

In the case put, and in fact in almost all
political contests, each question involved is dis-
cussed separately, and the conclusion pro-
fessedly drawn is simply the affirmative or the
negative of the particular question, as the case
may be; but the conclusion intended is, not
the affirmative or negative of the particular
question, but that of all of them taken to-
gether—thus presenting a case of irrelevant

Hence, generally, in political contests the
actual issue presented is simply as to the ascen-
dancy of one of two parties; while the
voters are persuaded, or persuade themselves,
that they are deciding some other issue. Hence
it results—as a general though not as a uni-
versal proposition—that politics becomes a
mere struggle for political supremacy.

§ 173. IRRELEVANT CONCLUSION.—All fal-
lacies of judgment must, as we have observed, take the form of irrelevant conclusion (§ 132); which, in turn, becomes a fallacy only when used as an equivalent to some other proposition. Hence the examples of fallacy already given, and many of those to be given hereafter, will equally serve our present occasion.

**Examples**

§ 174. The Doctrine of Absolute Sovereignty.—The use made of this doctrine by its advocates presents a conspicuous example of this fallacy. The doctrine, like all other nonsensical theories, is in itself innocuous, and becomes otherwise only by illicit use. But it is invariably used in some different and significant sense, as, *e. g.*, Rousseau's theory of the "Sovereignty of the People," which gave rise to the various political doctrines rife in the French Revolution, and to which historians have ascribed the terrible scenes of the Reign of Terror; from which they draw the inference that it is dangerous to apply Logic to practical politics. But this also is a case of Irrelevant Conclusion. For the conclusion should be only that Fallacy is dangerous, *i. e.*, not Logic, but the want of Logic.

§ 175. Sovereignty of the Law.—Of the various forms of the doctrine of Sovereignty, that of the Sovereignty of Right, or the Law,—
as it metaphorically expresses a doctrine at once true and fundamentally important,—might seem to be unobjectionable were it not that, in the direct effect of its language, it is merely nonsensical, and therefore liable to be used as equivalent to some other form of the doctrine, as, e. g., in the use made of it by Von Holst in his *Constitutional History of the United States*; where his expressed conclusion is that "Sovereignty is One and *Indivisible*—the Sovereignty of the Law." But his real doctrine—to the establishment of which all his arguments are marshalled—is that sovereignty is indivisible, and therefore vested exclusively, not in the law, but in the Federal Government, and not to any extent in the States.

§ 176. *Austin's Use of the Doctrine of Sovereignty.*—An example of this fallacy is furnished by Austin and his followers in the use made by them of their conclusion, that "*Sovereign power is incapable of legal limitation*"; which, accepting his definition of the law as being merely an expression of the will of the sovereign, is quite true, and altogether innocent; for obviously one's power cannot be said

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1 This—though, if the sense of the term be observed, a harmless proposition—is not a very consistent one; for, as in the United States, each State, as well as the Federal Government, has its own independent system of law, it would seem to follow that there are several sovereignties.
to be limited by his own will; but the proposition is habitually used in the ordinary sense of the terms.

§ 177. **USE OF THE DOCTRINE BY HOBBES.** —Another example, precisely similar, is furnished by Hobbes, who logically deduces from his premises the conclusion that "the right or just power" of the sovereign over the life and fortunes of the subject is unlimited; and the corresponding duty of the subject, absolute; which, according to his definition of the terms, *right, justice, and duty*, means simply that the so-called right of the sovereign is an unbridled or lawless power, to which prudence demands of the subject that he should submit for fear of worse consequences. The conclusion, in the sense of the terms defined, is, therefore, quite true; but it is habitually used by him and by modern English jurists as though the terms, *right, justice, and duty* were defined in their ordinary and proper sense.

§ 178. **BENTHAM’S MISUSE OF THE THEORY OF PRIVATE UTILITY.** —But the most flagrant example of this fallacy is that of Bentham, who, having established, or professed to have established, the doctrine of *Private Utility*, or *Utility to the Individual*, —which asserts that the sole possible motive of human conduct and the only standard of right and wrong is self-interest,—afterwards assumes as equivalent to it the
principle of General Utility, and systematically uses the latter as the premise established.

§ 179. Misuse of the Theory of General Utility.—This theory, in the use habitually made of it by Bentham and by utilitarians generally, also presents a most instructive example of this fallacy. The theory, being non-significant, is in itself innocuous; but it is commonly used as equivalent to the proposition that the interest of the majority is the sole test of right, or, as expressed by Bentham, "as equivalent to the sacred truth that the greatest good of the greatest number is the foundation of morals and legislation." Thus we have the apparently innocuous principle of General Utility converted into the execrable maxim that the good of the majority is alone to be consulted.

§ 180. Bentham's Defence of Usury.—Bentham's celebrated defence of usury has been commonly regarded ever since its publication as finally settling the question involved; but in fact it presents a striking example of the fallacy of Ignoratio Elenchi.

His thesis, as proposed, is to establish "the liberty of making one's own terms in money bargains"; and his conclusion, which is entirely legitimate, is that no man, not under disability, "ought to be hindered, with a view to his own advantage, from making such bargains in the
way of obtaining money as he sees fit." But obviously this is to mistake the issue; for the question is, not whether one should have the liberty of making usurious contracts, but whether he should be compelled to perform them (§ 167), and hence his conclusion is obviously irrelevant. He fails, therefore (though the world has thought differently), to establish his proposition.¹

§ 181. SPENCER'S ARGUMENT.—Spencer's argument—in Social Statics and Justice—for liberty of contract is also an example of the same fallacy. His first principle is his well-known law of equal liberty, namely, "that every man is free to do that which he wills, provided that he infringes not the equal freedom of any other man." From this principle he deduces, with admirable logic, the several personal rights that may be summed up in the general right of self-ownership, and also the right of property, and, as a corollary to the last, the right of free exchange, and from that (illogically, § 189) the right of free contract; but he illicitly assumes, with Bentham, that the

¹ In these observations it will be understood we are considering, not the moral or political question as to the propriety of enforcing contracts for the payment of interest (on which we have nothing to say), but simply the logical question as to the validity of an argument in favor of usury that has served to convince mankind of its righteousness, and that is universally regarded by an unlogical world as conclusive,
question is one touching the liberty of contract, and not as to the righteousness of coercing the parties (§ 167), which was his thesis. Hence his conclusion is essentially distinct from the real conclusion intended, which is, that men should be compelled to perform contracts.

§ 182. BERKELEY'S THEORY AS TO THE NON-EXISTENCE OF MATTER.—This furnishes another example. His argument is that, if matter exists, it is impossible for us to know the fact, or to know anything about it. But this conclusion he habitually uses as equivalent to the proposition that "matter, in fact, does not exist," i. e., he substitutes the "non-existence of matter" for "ignorance of its existence."
CHAPTER XIII

ILlicit CONVERSIONS

§ 183. Simple Conversion of Universal Affirmative Proposition.—The most usual form of this fallacy occurs in the simple conversion of a universal affirmative proposition, as, e.g., where from the proposition "Y is X" we illicitly infer that "X is Y"; and to this form all other cases may be reduced. The fallacy is so obvious that it might be supposed it could not often occur, but it is in fact very common.

Examples

§ 184. Confusion of Proposition with Judgment.—An example of it seems to be presented by the commonly received doctrine that "a proposition is a judgment expressed in words"; which seems to result from an illicit conversion of the proposition that a "judgment expressed in words is a proposition."

§ 185. Illicit Conversion by Negation.
—The fallacy frequently occurs in the conversion of a proposition by negation or contradiction. Thus, e. g., the proposition "Y is not X" becomes by negation "Y is not-X"; from which—converting per accidens—we may infer that "Some not-X is Y"; but not—as is often inferred—that "All not-X is Y."

By this method any universal affirmative proposition ("Y is X") may be converted into a proposition between the negatives of its terms (i. e., Not X is not Y); but not, as is often done, without converting the terms,—i. e., from the proposition "Y is X" we may infer that "Not X is not Y," but not that "Not Y is not X" (§ 91).

§ 186. An Argument of Hobbes.—A striking example of this fallacy is presented by Hobbes, that prince of logicians. Justice he defines as the keeping of covenants, and injustice as the failure to keep them. But, according to his theory, covenants become valid only upon the institution of government, from which they derive their validity. Hence in a state of nature there is neither justice nor injustice. But he says also: "Whatever is not unjust is just," and this conclusion—which is contradictory to his main position—is obviously arrived at by an illicit conversion of the universal affirmative proposition, "Whatever is just is not-unjust."
CHAPTER XIV

ILlicit Substitutions of Terms

§ 187. Substitutions of terms may consist either in the substitution of a new vocable or vocal sign, or in the substitution of a new sense to the same vocable. The latter is always illicit, and constitutes the Fallacy of Equivocation. The former will be considered in this, the latter in our next chapter.

The substitution of new terms of equivalent signification for terms originally occurring is the most common and extensive in application of all the processes involved in ratiocination; and the corresponding illicit processes—if we include equivocation—may be regarded as including all fallacies whatever. Hence the examples already given, and especially those given under the head of Irrelevant Conclusion, will serve equally well to illustrate the fallacy now under consideration.

Examples

§ 188. Austin’s Argument.—Many examples of this fallacy are furnished by Austin,
as, e. g., in substituting for the predicate of the proposition that “The sovereign power is incapable of legal limitation,” the term “legally despotic,” and thus inferring from the former proposition that government is vested by law with despotic power; which is not only untrue, but upon his own theory impossible. For, if law is but an expression of the will of the sovereign, it is equally absurd to say either that the sovereign power “is limited” or that “it is conferred” by law.

§ 189. Spencer’s Argument.—Another example is furnished by Spencer in inferring from the “right of free exchange” the “right of free contract,” which is in effect to substitute genus for species in the subject of a universal affirmative proposition. For exchange is only a species of contract (v. supra, § 181). It is true that the right of free contract cannot be doubted, but the substitution is none the less a logical fallacy.

§ 190. Fletcher vs. Peck.—Still another example of this fallacy is furnished by Chief-Justice Marshall (the greatest and most logical of American jurists) in Fletcher vs. Peck, 6 Cranch, 135; where it was decided that an act of the Legislature of Georgia revoking a grant of land was in contravention of the provision of the Constitution of the United States forbidding the States to pass any act “impairing
the obligation of contracts.” The argument in effect was that a grant is a contract, and that this was impaired by the act; which was in effect to substitute “Contract” for “Obligation of Contract.” The fallacy is the more glaring from the fact that a grant is an executed contract, which carries with it no obligation. Hence the constitutional provision must be held to refer only to executory or obligatory contracts.
CHAPTER XV

EQUIVOCATION

§ 191. The ambiguity of terms and sentences (Homonymia et Amphibolia) is undoubtedly the most prolific of all sources of fallacy. This is recognized by all logicians, and, indeed, by philosophers generally; but we doubt that many appreciate the extent of the evil or the universality of the danger to which men are exposed by reason of it, or (especially) their own infirmity in this respect.

"Instances of this fallacy," says Mr. Mill, "are to be found in most all the argumentary discourses of unprecise thinkers"; a proposition true in its literal statement but false in its obvious implications; for it implies that the proposition is not true of precise thinkers, and also (though with becoming modesty) that it is not true of the author. But in fact the most precise, or, as we would prefer to say, the most logical thinkers are liable to fallacy, and especially to this kind of fallacy; and none
more so than Mr. Mill. In this respect, if fallacies be regarded as intellectual sins, we may say: "There are none righteous. No, not one." For it is with logicians as with generals: the best that can be said of them is, that the greatest are those who commit the fewest blunders. Hence the only difference, other than degree, between the more precise or logical thinker and the unprecise is, that the fallacies of the latter are difficult, those of the former easy to expose. Hence it may be said that, while it is the greatest achievement to be right, it is no mean achievement to be clearly and unequivocally wrong, i. e., perspicuous in our errors. Hence the value of the political theories of Hobbes and Austin, the most logical of modern writers; which, though false, and even pernicious, are yet full of instruction.

Nor is the proportion of men of great logical genius so large as is generally supposed. They are in fact as scarce as great generals, or great statesmen, or great poets. Nor is it to be assumed that philosophical writers are less liable to this and other fallacies than the less pretentious classes. "For it is most true, as Cicero saith of them somewhere, that there can be nothing so absurd but may be found in the books of the Philosophers" (Hobbes, Lev., chap.

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1 This is very fully shown by Mr. Jevons (Pure Logic and Minor Works, p. 201).
v.). So, as observed by the author cited, the educated classes generally are inferior to the vulgar in this respect. For "those men that take their instruction from the authority of books, and not from their own meditations, [are] as much below the condition of ignorant men as men endued with true science are above it. For between true science and erroneous doctrines, ignorance is in the middle" (Id., chap. iv.). Hence no one should imagine himself free from this general infirmity of mankind; and he who most thoroughly realizes his weakness in this respect may, like Socrates, be justly pronounced the wisest of mankind. All are liable to it; and he who supposes he is not is simply unaware of his infirmity.

The nature of the Fallacy of Equivocation is obvious, and has been sufficiently explained. It remains, therefore, only to illustrate it by appropriate examples, and for this purpose the examples already given under other heads will—with one or two others—be sufficient to serve our purposes.

**Examples**

§ 192. Equivocal Use of Nonsensical Terms.—Some of the most important cases of this fallacy occur from the use of nonsensical terms. The very nature of these is that they
cannot be used for any practical purpose, except by changing their meaning and thus giving them a definite sense; and hence, for the propositions in which they occur, significant propositions are always substituted. Thus, as we have seen, the term Sovereignty varies essentially in meaning, as used in the several doctrines of Personal Sovereignty, Corporate Sovereignty, the Sovereignty of the People or State, and the Sovereignty of Right or the Law; all of which different senses of the term are inconsistent with each other, and all, except the first, in their direct sense, without definite signification, or, in other words, nonsensical. Yet the term is habitually used by political writers without distinguishing the sense in which it is used, or without attempting to give it any definite signification. But in the practical application of the doctrine of Sovereignty the term is invariably used as equivalent to such definite conclusions as the occasions of the writer may require, or as a premise from which such conclusions may be deduced; and thus the most extravagant doctrines are apparently established. Of which, as we have seen, a striking example is furnished by Prof. Von Holst (§ 175); and others equally appropriate may be easily collected from almost any work touching the subject.

The same observation will apply to the
theory of general utility, or Utilitarianism, and also to the notions that the will of the government is the *united will of the people*; that the State is an *Organism*; that it is founded on compact, etc.; all of which are, in their direct sense, in themselves *nonsensical*, and therefore innocuous, but are habitually used as premises to establish all sorts of extravagant conclusions.

§ 193. **Of Equivocation Generally.**—The above will suffice for examples of equivocations consisting in giving significance to *nonsensical* terms. In illustrating other equivocations, the only embarrassment consists in the number of examples that crowd upon our attention; but the following may be sufficient.

§ 194. **Argument of Austin.**—One of the most striking of these is furnished us by the argument of Austin in support of his famous position that judicial decisions are in their essential nature *laws* or statutes, and the judges, in fact, legislators; and another by his equally remarkable position that "*Custom does not constitute part of the law*"; both of which rest upon the equivocal use of the ambiguous term "*Law*"; which may denote either a *law* or *statute* (*lex*), or the *Law* (*Jus*).

§ 195. **An Argument of Bain.**—An extremely effective example of this fallacy is also
furnished by Mr. Bain in his statement of the doctrine of Utility. It consists in using the term "party" in the double sense of a natural and of a corporate person. Utility, he says, is "the tendency of actions to promote the happiness and prevent the misery of the party under consideration; which party is usually the community in which one's lot is cast." ¹

§ 196. AN ARGUMENT ATTRIBUTED TO PROFESSOR HUXLEY.—Still another example is presented by an argument attributed to Professor Huxley. It consists in the equivocal use of the term "power," which is commonly used in two senses, namely, as denoting actual power, or might, and as denoting rightful, or jural, power, or right. The argument is as follows: "The power of the State may be defined as the resultant of all the social forces within a definite area. It follows, says Professor Huxley, with characteristic logical thoroughness, that no limit is or can be set to State interference" (A Plea for Liberty, Donisthorpe).

This fallacy is common to all the Austinian school of jurists, and, indeed, constitutes the common fundamental infirmity of all their disquisitions. These jurists, according to their theory, have, indeed, no right to use the term in any but the former sense; but, as we have

¹ Bentham is guilty of the same fallacy (Principles of Legislation).
seen, after establishing their conclusions they habitually use it as though equivalent to *right*, in the proper sense—a notion that can properly have no place in their system.
CHAPTER XVI
THE TRADITIONAL DOCTRINE OF FALLACIES

I

ARISTOTLE'S CLASSIFICATION OF FALLACIES

§ 197. The received classification of fallacies,—adopted by the schoolmen from Aristotle,—though remarkable for its profound insight, has but few pretensions to scientific accuracy; and it is to be suspected that much of the obscurity and confusion that surround the subject results from the undue authority given to it by logicians. It has, however, so profoundly affected logical doctrine and nomenclature that, apart from its intrinsic value, it must always remain one of the principal subjects for the student's attention.

§ 198. TABLE OF FALLACIES.—According to this scheme, fallacies are divided into two classes, called by the schoolmen and by later logicians, Fallacies in Dictione, or in Voce (i. e., in diction or speech), and Fallacies extra Dictionem, or in Re (i. e., not in diction, but in
matter). Of the former class six forms or examples are given, and of the latter, seven, which are as follows:

Aristotle’s Division of Fallacies

I. Fallacies in Dictione:
(1) Homonymia (Ambiguity of Terms).
(2) Amphibolia (Ambiguity of Sentence).
(3) F. Compositionis (F. of Composition).
(4) F. Divisionis (F. of Division).
(5) F. Accentus (F. of Accent).
(6) F. Figurae Dictionis (F. of Figure of Speech).

II. Fallacies Extra Dictionem:
(1) F. Accidentis (F. of Accident).
(2) F. a Dicto Secundum Quid ad Dictum Simpliciter (Illicit Substitution of Unqualified for Qualified Terms).
(3) Ignoratio Elenchi (Irrelevant Conclusion).
(4) F. Consequentis (Non-Sequitur).
(5) Petitio Principii (F. of Illicit Premise).
(6) Non-Causa pro Causa (Mistaking Cause).
(7) F. Plurium Interrogationum (F. of Several Issues in One).

§ 199. Observations upon this Classification.—As will be seen presently, all the fallacies In Dictione are simply cases of Equivocation, and of the fallacies Extra Dictionem all except the 4th (F. Consequentis) are Fallacies of Judgment; under which head most of them have already been considered at large. The
excepted fallacy (the *F. Consequentis*) includes all the Fallacies of Inference, except *Equivocation*. It is obvious, therefore, that the current expressions (*In Dictione* and *Extra Dictionem*)—whether from being a mistranslation of Aristotle's language or otherwise—do not truly express the nature of the distinction between the two kinds of fallacies, and are, therefore, calculated to mislead us—as they have Whately and others—with regard to it.

§ 200. The true scheme of division is as follows:

*Table of Fallacies*

I. **Fallacies in Dictione** (Equivocation).

( Including the six forms specified in the first table.)

II. **Fallacies Extra Dictionem**.

(1) **Fallacies of Judgment**.

( Including all fallacies *Extra Dictionem* given in the table, except *F. Consequentis*.)

(2) **F. Consequentis** (*Non-Sequitur*).

( Including all Fallacies of Inference except Equivocation.)

(a) **Formal** Fallacies (*i. e.*, of Inference).

( Including Undistributed Middle, Illicit Process.)

(b) **Material** Fallacies.

( Including Illicit Substitutions of New Terms.)
The terms "Formal" and "Material Fallacies" correspond to the "Logical" and "Material Fallacies" of Whately, whose "Semi-logical Fallacies" correspond precisely to the fallacies In Dictione of Aristotle, or, in other words, to the Fallacy of Equivocation. This division of Whately's has, since his time, been very generally adopted; but, as is remarked by Mansel, it "is not the ancient principle of distinction which is stated with more or less clearness by several logicians," as, e.g., in the following definitions of Sanderson: "Every fallacy In Dictione arises from some ambiguity (multiplicitate) of expression." "Fallacies Extra Dictionem are those in which the deception happens, not so much from some ambiguity latent in the words themselves, as from ignoring things" (i.e., the notions expressed). "The former arise," says Mansel, "from defects in the arbitrary signs of thought, and hence are generally confined to a single language, and disappear on being translated into another. The latter are in the thought itself, whether materially, in the false application of notions to things, or formally, in the violation of the laws by which the operations of the reason should be governed; and thus adhere to the thought in whatever language it may be expressed. Under this head are thus included both false judgments and illogical reasonings."
LOGIC

(i.e., both Fallacies of Judgment and Fallacies of Inference) (Mansel’s Aldrich, p. 132).

II

FALLACIES IN DICTIONE (EQUIVOCATION)

§ 201 (1) (2). HOMONYMY AND AMPHIBOLY.—These are both cases of the Fallacy of Equivocation, the former consisting in the illicit use of ambiguous terms, the latter in the illicit use of ambiguous sentences. They are essentially of the same nature; and we, therefore, as is most in accord with the usage of our language, class them together under the common name of Equivocations. This fallacy has already been fully considered.

§ 202 (3) (4). COMPOSITION AND DIVISION.—These fallacies are essentially of the same nature. They consist in using a term successively in a distributive and in a collective sense, or, in other words, in substituting for a term used distributively the same term used collectively, or vice versa. The former constitutes the Fallacy of Composition, the latter the Fallacy of Division.

The following are examples of the Fallacy of Composition:

3 and 2 (distributively) are two numbers;
5 is 3 and 2 (collectively);
∴ 5 is two numbers.

He who necessarily goes or stays (i.e., either
necessarily goes, or necessarily stays) is not a free agent;

But every one either necessarily goes or stays (i. e., necessarily does one or the other);

∴ No one is a free agent.

The following are examples of the Fallacy of Division:

5 is one number;
3 and 2 (collectively) are 5;
∴ 3 and 2 (distributively) are one number.

The angles of a triangle are equal to two right angles;
A B C is an angle of a triangle;
∴ A B C is equal to two right angles.

All the black and white horses of the deceased (i. e., all the black, and all the white horses) are the property of the legatee;

The piebald horses are black and white (i. e., each is black and white);
∴ The piebald horses are the property of the legatee.¹

Obviously these fallacies (Composition and Division) constitute merely a species of equivocation, i. e., of either Homonymy or Amphiboly.

¹ The last example is suggested by the celebrated Moot case of the legacy of "all the testator's black and white horses." The question was, whether the legatee was to have the black and the white horses, or the piebald horses, i. e., the horses that were each black and white. The legatee claimed that he was entitled to both classes; and, hence, in the one or the other of his claims, was guilty of this fallacy.
§ 203 (5). The Fallacy of Accent or Prosody (F. Accentus F. Prosodīē).—This fallacy is also a species of equivocation, i.e., either Homonymy or Amphiboly. It consists in varying the meaning of a term or proposition by change of accent, tone, or punctuation.

The most extreme case of this is that of *irony*, by which the sense is precisely reversed, as, e.g., in the speech of Job to his friends: "No doubt but you are the people, and wisdom shall die with you." In this way, i.e., by ironical use afterwards forgotten, the name of the subtle doctor, Duns Scotus, has come to be the peculiar name of a fool (i.e., dunce). The fallacy resulting from changing the sense of an ironical expression is too obvious to be dangerous, but if it should occur would be a case of *F. Figure Dictionis*.

§ 204 (6). Figure of Speech (F. Figurāē Dictionis).—This fallacy (which is also merely a species of equivocation) consists in the illicit use of figures of speech, or, in other words, in substituting for the *indirect* or *figurative*, the *direct* or *literal* sense, as in the following example:

"Herod is a fox;
A fox is a quadruped;
\therefore Herod is a quadruped."
Or as in the following example, which was given by a student called on for a syllogism. The logical Professor, it may be explained, was of corpulent habit, and known as "Old Boll."

"All flesh is grass, the Scriptures say,
And grass when cut is turned to hay;
Now if Death's Scythe Old Boll should take,
Golly! What a haystack he would make!"

But more serious examples may be found among those already given, as, e. g., the equivocal use of the term power in the argument attributed to Professor Huxley, and also in the misuse of the propositions that "the State is a person," that "it is an organism," that "its will is the united will of the people," that "it has an interest or welfare distinct from that of the people," etc., as heretofore explained. A striking example of this fallacy is also presented in the famous case of Dartmouth College vs. Woodward (§ 137). The fallacy consisted in regarding the college as a person; which was only figuratively true. For a corporation is a quasi-person only, i. e., is regarded as a person for certain purposes only.

§ 205. Hamilton strangely speaks of this as "a contemptible fallacy," and—as though to furnish an example at once of confusion of
things essentially different and of misapprehension of the nature and scope of Logic—he couples with the Fallacy of Figure of Speech that of *Equivocation*, as being, the latter, a species of the former, instead of *vice versa*, as is in fact the case. "These fallacies," he says, ('*sophismata equivocationis, amphibolæ, et accentus*) may easily be reduced to *sophismata figuræ dictionis*; they are only contemptible modifications of this contemptible fallacy."

But, as is in effect observed by the author to whom we are indebted for the above quotation, when we reflect that nearly all words denoting mental or moral qualities or acts—which is but to say nearly all terms used in the different branches of the science of human nature—are in their origin metaphors, derived from sensible objects or events as, *e.g.*, *intuition, perception, apprehension, inference, induction, deduction, reflection, education, justice, right, wrong, straight, power, organic*, etc., and that these terms still carry with them, to a large extent, their material associations, by which, as the history of philosophy shows, we are continually being misled, we can hardly fail to agree "that the sophism *Figuræ Dictionis*, so far from being contemptible, is worthy of our closest and most watchful consideration" (*Theory of Thought*, Davis, p. 27).
§ 206. Observations.—Of these fallacies, all except the fourth are Fallacies of Judgment; and four of them, namely, *Ignoratio Elenchi*, *Petitio Principii*, *Non Causa pro Causa*, and *F. Plurium Interrogationum*, have already been considered in detail under that head. The others, namely, the Fallacies of *Accident*, of *Secundum Quid*, and of the *Consequent*—of which the first two are also Fallacies of Judgment—remain to be considered.

Logicians are widely at variance with reference to the nature of these fallacies; and, if we may judge from the translations and from the confusion reigning over the subject, Aristotle’s own explanation of them must be regarded, in some particulars, as hopelessly obscure. Hence, though I have attempted to interpret his meaning correctly, I am by no means sure that I have succeeded in this better than others. It may, however, be claimed for the exposition of the subject here given that it is at least intelligible and consistent, and that, in connection with the rest of Aristotle’s scheme, it renders his classification of the fallacies complete. And, it may be added, it is in accord with the best authorities.
§ 207. The Fallacy of Accident (F. Accidentis).—This fallacy has its source in the assumption that an accident of some of the significates of a term, or of all its signicates for a certain time, is an accident of the term, and therefore predicable of it without qualification (v. supra, § 49.) This assumption in the case of an inseparable accident of all the signicates of the term is, indeed, legitimate; for obviously such an accident may always be predicated of all the signicates of the term, and hence of the term. But with separable accidents of the signicates of a term, it is otherwise; for, though these are commonly spoken of as accidents of the term, they are not such in fact, for their relation to the term is temporary or transient.¹ Hence such an accident can be predicated of the term only for so long as it continues to be an accident of it, or, in other words, only with relation to some particular time expressed or understood. For in the logical proposition the copula has no relation to time, but expresses simply a permanent significative relation between the terms, and

¹ The terms separable and inseparable accidents can apply only to real individuals, and hence only to concrete terms or terms of first intention. With relation to these the distinction is sufficiently obvious. Thus, e. g., with reference to Socrates, "Stagyrite" is an inseparable accident; "standing," "sleeping," etc., separable—the last being predicable of him only at times.
hence a separable accident cannot be predicated generally of a term. For, as is said by Aristotle, "it is uncertain when [i.e., at what times] an assertion can be made of a thing present from accident"; or, in other words, whether at any given time the accident continues to exist (Soph. Elench, chap. xxiv.). Thus, e.g., an attacking party might be rightly informed at a given time that the enemy was sleeping, and hence conclude that it would be safe to attack him; but it might be a fatal error to assume the truth of the premise as continuing to exist an hour later.

§ 208. Definition of the Fallacy.—The fallacy may therefore be defined as consisting in predicking of a term a separable accident of its significates without qualifying it by referring to the time at or during which it is inherent; or, in other words, in assuming, in place of a proposition of which the predicate is an accident thus qualified, another proposition of which the predicate is the accident unqualified; as if, e.g., from knowing a man is lame we should assume that he is permanently lame. Or the subject may be more generally illustrated as follows: Let Y denote the subject ("John"), A the accidental predicate ("temporarily lame," i.e., "lame for the time being"), and X the general predicate ("permanently lame"); then we may be entitled to say "Y
is A"; but to assume, in place of this, that Y is X would be to substitute for A the term X, i.e., species for genus in the predicate of a universal affirmative proposition. For the class of "temporarily lame" will include all the "permanently lame," and many others.

It will be noted here that there is necessarily a significative relation between the accidental and the general predicate, namely, that of partial coincidence. Hence, to substitute X for A is, in effect, to substitute AX (i.e., "Some A") for A, which presents a case of illicit substitution of species for genus in the predicate of an affirmative proposition.

It will also be observed that the Fallacy of Accident is defined as consisting in the illicit assumption of a premise. But, where the same fallacy occurs in a formal inference, it constitutes the Fallacy of Undistributed Middle, which is a case of Non-sequitur or F. Consequentis, as may be thus illustrated:

\[
\text{Some A is X} \\
\text{Y is A} \\
\therefore \text{Y is X}
\]

The stock example of this fallacy, which I have taken from Aldrich, is as follows:

"What you have bought you have eaten; you have bought raw meat; therefore you
have eaten raw meat" (Quod emisti comedisti; crudum emisti; ergo crudum comedisti); which may be expressed in the following syllogism, which, in form, is unobjectionable:

The meat you buy is raw;
The meat you eat is the meat you buy;
∴ The meat you eat is raw.

The fallacy here may be regarded as a case of equivocation, consisting in the use of the term "raw" in the major premise in the sense of "raw when bought," and in the conclusion in the sense of "raw when eaten." But if the term "raw" be construed simply in both cases (i. e., as used without qualification), the fallacy must be regarded as a case of F. Accidentis, consisting in the illicit assumption of the major premise. For all that can be rightfully affirmed is that the meat bought is raw at the time of purchase; instead of which it is assumed that it is permanently raw. For, as we have observed, in the logical proposition the copula includes both the future and the past, and the significative relation between the terms is asserted, not as true only at the moment of assertion, but before and afterwards; and hence a universal proposition may always be negatived by showing an instance to the contrary, either in the past or in the future.
The following examples are furnished us by Aristotle, and are given as paraphrased in the notes of Mr. Owen's translation:

"Do you know what I am about to ask? No. But I am about to ask whether virtue is good. Therefore, you know not whether virtue is good."

"Do you know who approaches? No. But Socrates approaches. Therefore, you do not know Socrates."

Here in each case the most obvious source of the fallacy is in the use of the equivocal terms, "What I am about to ask" (in the first case), and "Who approaches" (in the second). But this ambiguity may be removed and the arguments expressed syllogistically in unobjectionable form as follows:

(1) The question, I am about to ask, is unknown to you.
   The question whether virtue is good is the question I am about to ask.
   .: The question whether virtue is good is unknown to you.

(2) The man approaching is unknown to you.
   Coriscus is the man approaching.
   .: Coriscus is unknown to you.

Indeed, even as thus expressed, the most obvious solution of both these fallacies is still to regard them as cases of equivocation, con-
sisting in using the term "unknown to you" in a double sense, i. e., in the major premise in the sense of "unknown to you before you are told," and in the conclusion in the sense of "unknown to you after you are told." But if the term be regarded as used in the same sense in both places, the case is evidently one of F. Accidentis, consisting in the illicit assumption of the major premise, or, in other words, in the illicit substitution of the unqualified term, "unknown to you," for the qualified term, "unknown to you before you are told," which alone was admissible as a predicate.

§ 209. The Fallacy of Secundum Quid (F. A Dicto Secundum Quid Ad Dictum Simpliciter).—This fallacy consists in assuming an unqualified in place of a qualified proposition. But as the copula has but one meaning, a proposition can be qualified in no other way than by qualifying one or both of its terms. Hence the fallacy must consist in substituting for an unqualified a qualified term.

But a term can be qualified (i. e., its signification or extension altered) only by coupling with it another term that partly, but not wholly, includes it, thus making a new term of less extension, as, e. g., men by white, which gives us for the new term, white men; or, more generally, Z, Y, or X, by A, which gives us, for new terms, AZ, AY, and AX, all
included in, but of less extension, than the originals; or, in other words, the class denoted by a qualified term will always be a species of the class denoted by the unqualified term. Hence the Fallacy of *Secundum Quid* is simply a particular case of the illicit substitution of genus for species in the subject of an affirmative, or in either the subject or predicate of a negative proposition.

Where the illicit substitution occurs in the inference, the fallacy belongs to the general class of fallacies that go by the name of *F. Consequentis* or *Non-sequitur*; but if in one of the premises, it constitutes the Fallacy of *Secundum Quid*, now under consideration; which must, therefore, like the *F. Accidentis*, be regarded as a case of Illicit Assumption of Premise, or of *Petitio Principii*. The Fallacy of *Secundum Quid* may therefore be defined as consisting in the illicit assumption of a premise in which there is an unqualified term in place of another in which the same term is qualified; or, as expressed by Aristotle, is assuming that "what is predicated in part is *spoken simply*" (Soph. Elench., chap. v., 2).

§ 210. Of the Relation between the Fallacies of Accident and Secundum Quid.—The Fallacy of *Secundum Quid* will therefore include the Fallacy of Accident, which is but a particular case of it. Or, in other
words, the latter is a species of the former, its specific difference being that the qualification omitted relates exclusively to time; whereas, in the case of Secundum Quid generally, the omitted qualification may relate either to time or to place, quantity, or any other quality or attribute.

The following examples of the F. Secundum Quid are taken from various sources:

(1) Pernicious things are things to be forbidden;
    The use of wine is pernicious;
    Therefore the use of wine is a thing to be forbidden.

(2) Things productive of bad effects are unfit for use;
    Antimony is a thing productive of bad effects;
    ∴ Antimony is unfit for use.

(3) Things productive of bad effects are to be discouraged;
    Eloquence is a thing that produces bad effects;
    ∴ Eloquence is to be discouraged.

(4) Things destructive to human life are to be avoided;
    Medicine is a thing destructive to human life;
    ∴ Medicine is to be avoided.

(5) Y is X
    Z is Y
    ∴ Z is X.
In each of these arguments—all of which are regular in form—the fallacy consists in the illicit assumption of the minor premise, consisting in substituting in the subject an unqualified in the place of a qualified term, viz., in the first, the term “use” for “excessive use”; in the second, “antimony” for “antimony when misapplied”; in the third, “eloquence” for “eloquence when abused”; in the fourth, “medicine” for “medicine when used by ignorant doctors”; and in the fifth,—denoting by $A$ any term qualifying $Z$,—$Z$, for $AZ$. The fallacy, therefore, in each case consists in the substitution of genus for species in the subject of an affirmative proposition, and hence differs from the corresponding fallacy of inference simply in being an illicit assumption instead of a formal inference.

§ 211. Erroneous Views of Logicians as to these Fallacies.—The *F. Accidentis* was defined by Aldrich, and probably by the old logicians generally, as in the text. But Whately, who is followed by most of the later logicians, defines it as the converse of the Fallacy of *Secundum Quid*; and since then the subject has been involved in the greatest confusion. The prevailing view is thus expressed by De Morgan:

“(1) *The Fallacia Accidentis* and (2) that *a dicto secundum quid ad dictum simpliciter.*
The first of these ought to be called that of *a dicto simpliciter ad dictum secundum quid*, for the two are correlative in the manner described in the two phrases. The first consist in inferring of the *subject* with *an accident* that which was premised of the *subject only*, the second in inferring of the *subject only* that which was premised of the *subject with an accident*" (*Formal Logic*, p. 250).

The latter process is undoubtedly fallacious, but the former—*i. e.*, inferring of the *subject with an accident* that which was premised of the *subject only*; or, in other words, of inferring that what is predicated of a *term generally* may be predicated of the term as qualified by an accident—is entirely legitimate. For to qualify a term, either by an *accident* or otherwise, is simply to diminish its extension, and thus to create a subclass or species of the class denoted by the unqualified term; and according to the *dictum* whatever may be predicated of the *unqualified* term or *genus* may be predicated of the *qualified* term or species; or, in other words, in any universal proposition of which the unqualified term is the subject, the same term qualified by an accident may be legitimately substituted for it; that is to say, symbolically, denoting by *AY, Y* as thus qualified, if *Y* is *X*, then *AY* is also *X*; as may be thus illustrated:
In illustration of the supposed fallacy (*F. a dicto simpliciter ad dictum secundum quid*) De Morgan and others give us the story of the stork, from Boccaccio, which, as quoted by Professor Davis, is as follows:

"A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon the table the master desired to know what was become of the other leg. The man answered that 'the stork never had but one leg.' The master, very angry, but determined to strike his servant dumb before he punished him, took him the next day into the fields, where they saw storks standing each on one leg, as storks do. The servant turned triumphantly to his master, upon which the latter shouted, and the birds put down their other leg and flew away. 'Ah, sir,' said the servant, 'but you did not shout to the stork at dinner yesterday; if you had done so, he would have showed his other leg too.'"

The gist of which, the author says, "is the assumption that what can be predicated of storks in general can be predicated of roasted
storks,—a dicto simpliciter ad dictum secundum quid." But undoubtedly (assuming for the sake of the argument that dead and roasted one-legged storks belong to the genus stork) whatever may be universally predicated of storks may, unless the dictum be a delusion, be predicated of roasted and one-legged storks as well as of others. The error, therefore, consists, not in an incorrect inference of the particular proposition from the universal proposition including it, but in the illicit assumption of the universal proposition that whenever you shout at a stork it will put down a second leg, though it may have only one leg, and be dead and roasted.

§ 212. F. Consequentis.—There is much dispute as to the nature of the fallacy intended by Aristotle under this name. De Morgan and other logicians—following Aldrich—regard it as consisting in the "affirmation of a conclusion" which does not follow from the premises, or, in other words, as but another name for a Non-sequitur, which is at least the most convenient view.

§ 213. Classification of Fallacies of this Kind.—According to this view, the F. Consequentis will include (1) the merely formal fallacies, commonly known as fallacies of the syllogism; and (2) all the material fallacies of inference except Equivocation. The former
have been sufficiently treated in considering the rules of the syllogism; the latter, under the head of Substitution. The former as well as the latter, and also the fallacies of Equivocation (or In Dictione), are also, it will be remembered, fallacies of Substitution.
APPENDIX OF NOTES

A—§ 4

Perhaps, when men understand that the main sources of Philosophy are to be found in the study of words, we may hope to escape the dreary treadmill on which philosophers have hitherto been exercising themselves. All progress in Philosophy that has been made has been the result of the unconscious observation of this method—as, e. g., the work of Locke, which, though weak in its metaphysics, constitutes the greatest contribution to philosophy made in modern times; and which, as shown by Horne Tooke, is merely an essay on language. "Perhaps," he says, "it was for mankind a lucky mistake (for mistake it was) which Mr. Locke made when he called his book an Essay on the Human Understanding. For some part of the inestimable benefit of that book has, merely on account of its title, reached to many thousands more than, I fear, it would have done had he called it "A Grammatical Essay," or "A Treatise on Words or Language" (Divisions of Purley).

B—§ 6

Comparing the physical sciences and the mathe-
matics with the moral sciences, the latter are infinitely the more difficult of achievement; and also infinitely more important to the welfare of mankind. For under the name of the moral sciences are included all the several branches of the Science of Human Nature; which is obviously the principal concern of mankind, and as such the science to which all others are to be regarded as subsidiary. This was the distinguishing characteristic of Socrates' philosophy. It was expressed in the injunction written over the portals of the Delphic god: "Know thyself!" and in modern times has been finely rendered: "The proper study of mankind is man." It is also embodied in the fine old term, the *Humanities*, which signifies those parts of education that have for their end the development of our manhood or humanity, and which must therefore constitute the essential elements of a rational general education.

C—§ 8

This was the great discovery of Socrates; to the preaching of which, as the gospel most needed by men, his life was devoted. Nor have there been wanting, in succeeding ages, philosophers—and those the greatest—to continue his mission. But so averse are men to being convinced of their errors that nothing is more odious to them than the attempt. Hence, generally, all means of defence are regarded as legitimate,—that is to say, not only fallacies, but falsehoods and slanders, and, at times,
the prison, or the rack, or death. Thus Socrates was poisoned for this offence only; which, though otherwise atrocious, was creditable to the Athenians, as at least proving an uncomfortable mental susceptibility to the power of reasoning or Logic. For in modern times we have invented a better method of dealing with such fellows, and have developed a mental integument as impervious to the weapons of reason as that of the elephant or rhinoceros to the weapons of the primitive hunter; and against which the Socratic wit would batter in vain. Thus we are enabled to dispose of those who would disturb our mental peace and complacency, by simply refusing to listen to them, and by extolling our own idols,—like the Ephesians; who, in answer to the preaching of the apostles, "all with one voice, about the space of two hours, cried out: Great is Diana of the Ephesians." By these two means—which have been aptly called "the conspiracy of silence," and "the society of mutual admiration"—our opinions are now impregnably buttressed. Thus we live in a sort of Fools' Paradise; though, as Bacon says, "the apotheosis of error is the greatest evil of all, and when folly is worshipped, it is, as it were, a plague-spot upon the understanding" (Nov. Org., bk. i., aph. lxv.).

D—§ 11

The disuse of Logic must necessarily affect the teaching of Moral and Political Science, Metaphysics, and the Science of Human Nature generally;
for the investigation of which it is indispensably. Hence, as the proper study of mankind is man, it may be said that the universities of the day have fallen behind their predecessors in efficient performance of their most essential function. It should not be forgotten that the task of reorganizing European society as it emerged from the chaos of the dark ages was mainly effected by such men as Lanfranco, Suger, Anselm, and other churchmen—graduates of the mediæval schools and universities, and consequently educated in Logic and Law; studies the art of teaching which has been lost by our modern universities, and which yet surpass all others as means of a rational education. That this is the case with Logic, it is the aim of this work to show; with regard to the Law, the opinion of Burke, by those competent to judge, has been generally accepted,—that it "is one of the first and noblest of human sciences—a science which does more to quicken and invigorate the understanding than all other kinds of learning put together." Though, he adds, "it is not apt, except in persons happily born, to open and liberalize the mind exactly in the same proportion."

E—§ 12

The peculiar merit of Logic, as one of the Humanities, is its perfect cognoscibility, and the consequent facility with which it can be taught. Arnauld in the preface to the Port Royal Logic tells us that he undertook to teach a young noble-
man all that was useful in Logic in four days, and successfully performed the task. The claim is seemingly extravagant, but as his notion of Logic was confined mainly to the doctrine of the syllogism, and to so much only of the doctrines of the term and of the proposition as was incidentally necessary, and as the student was a young gentleman of remarkable ability, it may very well be credited. Nor will a more complete and comprehensive study of the subject add much to the labor of mastering it; if indeed it will not facilitate the task. The general diffusion of logical culture cannot be regarded, therefore, as a vain aspiration. The subject requires no preliminary culture other than the studies usually taught in the common schools, and may be readily mastered by almost any young man of average ability and the proper age—say sixteen or seventeen. And this will especially be the case with one who has thoroughly mastered the elements of algebra and geometry. Thus it is quite possible to devise a very brief course of study sufficiently thorough to train the student as a reasoning creature, and to make him equally competent with the graduates of our great universities to grapple with all the great problems of Politics and Morality; and, indeed, until our modern university education be reformed, even more so. This was illustrated by the mediæval universities, to whose graduates, as we have observed, the reorganization of society at the close of the dark ages was entrusted, and by whom the task was successfully accomplished; nor do I think
it extravagant to say that alongside of them in practical politics our modern graduates would be but children. Of the subjects taught outside of Theology the principal, as we have said, were Logic and Law, and these must be regarded as the most essential parts of a rational education. The latter will require long and persevering study, but a thorough logical training will render the student competent to master it; and without such training—either systemically taught to him at the outset, or gradually acquired in the study of the law itself—its mastery is impracticable; and the same observation is true with reference to Political Science generally.

F—§ 13

I have been admonished by a friend that the use of examples of this kind in an elementary work may be hazardous; and this, I understand, on the double ground that the younger student may find it difficult to understand them and the older, regard them as disputable; and that thus they must prove to the one a stumbling-block, and to the other foolishness. With regard to the last objection, it is to be admitted that if any of the examples are in fact disputable, the objection is well taken. But I am persuaded that, if they appear so to any one, it is only because of the universal bias of men in these unlogical times in favor of their opinions, and that any one who will provisionally reject all prejudice will see at once that the argument is in every case demonstrative. Or if in any case I am deceived,
then my own reasoning will serve for example. With regard to the younger student, the opinion seems to be that it would be better to illustrate the nature of the fallacies by the more familiar examples of the character commonly used in the current logics. But this, I think, to be a great mistake. The fallacies are themselves sufficiently simple to be readily understood, and trivial examples merely serve to lead the student to suppose that he is in no danger of falling into them. I have therefore thought it far better to take my examples from theories that have played and are now playing a great part on the stage of history. Nor are these, when treated logically, at all difficult, with a little reflection, to understand; and indeed it is to be assumed that, if a young man has arrived at the age at which he can study Logic profitably without some familiarity with these questions, his education has been much neglected. Neither this nor any part of my work can, indeed, be understood without the independent thought of the reader; but this also I consider not only a great advantage, but an essential condition to the right exposition of the subject. For though the principles of Logic are extremely definite, and therefore readily cognoscible, yet, as already observed, they require for their mastery the same kind and degree of study as is required by the mathematics; and there is no royal road to Logic any more than to geometry. If the student, therefore, will take the trouble to work out thoroughly these examples, and others of the same character (of which many will
suggest themselves), he will achieve not only a mastery of the principles involved in them, and of the practical use of Logic, that cannot be otherwise attained, but also an accurate, though limited, knowledge of all the great political, social, and moral questions involving the welfare of mankind; which, better than anything else, will serve as an introduction to those studies. I have also, in the use of these examples, another point in view, which is, that, by means of the application of logical principles, these apparently difficult problems are readily solved, and the most important heresies in Politics and Morality that afflict mankind exposed; and thus are proved, by practical illustration, the theses with which I commenced,—that in all the moral sciences the use of Logic is essential, and that the confused and unsatisfactory condition of the literature of these subjects is due to the decay of Logic.

In conclusion, however, I would say that while regarding the current examples used in the logics as inadequate for the illustration of the subject, I have not neglected them, but, in the chapter on the Traditional Doctrine of Fallacies, have confined myself mainly to them.

G—§ 14

This is strenuously objected to by Hamilton. "Dr. Whately," he says, "is contradictory. . . . In some places he makes the operation of reasoning not only the principal, but the adequate object of
Logic. . . . In others, he makes this total or adequate object to be the language. But as there cannot be two adequate objects, and as language and the operation of reasoning are not the same, there is therefore a contradiction" (Logic, i1).

But though language and reasoning are not the same, yet they are the same so far forth as Logic is concerned with either; for, as Logic has to deal only with reasoning expressed in language, it is necessarily concerned with both to the same extent; and we may say, with equal propriety, that the subject-matter of Logic is either language or reasoning.

The error of Hamilton lies in the illicit assumption that the term "language" is equivalent to the external logos, i.e., the expression, as opposed to the inward thought. But if language be construed as denoting both the thought and the expression, as it should be, the only objection disappears; and when thus construed, the proposition that Logic is concerned wholly with language is too clear to be disputed.

H—§16

The name given to the subject by Aristotle was the "Analytics." The name Logic seems to have been first applied to it in the time of Zeno, the Stoic. Many names have been invented to signify the scope of Logic,—as, e.g., the Architectonic Art; the Organon, or Instrument; the Ars Artium, or Disciplina Disciplinarum; Heuristic, or the Art of Discovering Truth; the Medicina Mentis, or the
Cathartic of the Mind, etc. (Thompson, Laws of Thought, § 35); and to these should be added the name given by Socrates to his own doctrine (which, though the fact is commonly overlooked, was nothing else than Logic), namely, the Obstetrics of the Mind (Maieusis).

Of these, the last two names express precisely the two main functions of Logic,—that is to say, 1st, to serve as a cathartic of the mind to rid it of the false persuasion of knowledge; for, as has been well said, "the natural state of the human mind" is "not simply ignorance, but ignorance mistaking itself for knowledge" (Grote's Plato, i., p. 373); and, 2d, to bring forth from the mind "answers of which it is pregnant" (Id., p. 367); or, in plain language, to develop and formulate the unformed ideas in our minds, whether innate or acquired from without. See Socrates' own account of this function, as given in the Theaetetus (Id., iii., p. 112).

I—§ 37

There is much confusion with modern logicians with regard to the nature of first and second intentions or notions, but the above definition seems to accord with the best authorities and expresses a distinction of fundamental importance. According to this definition, Notions of Second Intention will include all abstract notions, and also notions of classes of real individuals construed collectively; in which case they become abstract.

The following is the definition of Aquinas (Opus-
cula, cited Krauth, *Voc. of Phil. Art.*, ""Intention, First and Second "):

"Nouns of *first intention* are those which are imposed upon things as such, that conception alone intervening by which the mind is carried immediately to the thing itself. Such are *man* and *stone*. But nouns of the *second intention* are those which are imposed upon things not in virtue of what they are in themselves, but by virtue of their being subject to the intention which the mind makes concerning them, as when we say that man is a *species* and animal a *genus.*" Which seems to accord with our definition: that is to say, if we speak of *man* as denoting the class of individual men, the name is of the first intention, but if we regard *man* collectively as a significate of the class *animal*, the name is of second intention; and so with reference to all other abstract names. Names of second intention are precisely denoted by the term ""*universales a parte rei,“" — *i. e.*, universal notions considered apart from things, or, in other words, *abstract* notions,—and also by the term ""*beings of reason,” as quoted infra.

The division of names into names of first and of second intention was obviously intended to comprehend all names; and hence, if names of *first intention* are identical with *concrete* names,—as they evidently are,—names of *second intention* must include all *abstract* names; and it is not admissible to confine them (as Mansel does) to some of that class only. Accordingly a *universal* (*ens unum in multis*) is defined by Aldrich simply as a
predicable,—i.e., as "Nomen Commune, Univocum, Secundae Intentionis, uno verbo, Predicabilis, Sive Vox apta praedicare, i.e., Univoce dici de multis" (Ald. Log., p. 23).

It is singular that in the Port Royal Logic this distinction should be regarded as unimportant, and even made the subject of ridicule. "No one, thank God!" it is said, "now takes any interest in 'the universal a parte rei,' or 'beings of reason,' or in 'second intentions.' Thus, there is no ground to apprehend that any one will be offended at our having said nothing about them." But it may be safely said that no one can have an adequate conception, either of the nature or use of Logic, until the notion expressed in the term "second intentions," and the other phrases cited (which are similar in meaning), are thoroughly grasped.

Even where we use concrete terms, it is not the thing itself, but the notion of the thing that is present to the mind. For, as is said by Hobbes, "seeing names ordered in speech (as is defined) are signs of our conceptions, it is manifest they are not the signs of the things themselves." Hence, as Mansel says, "concepts (or notions) are the things of Logic." On this point Max Müller's Laws of Thought (the opening chapters) may be read with profit. For without acceding altogether to his theory,—that thought is impossible without language,—this is certainly true (ex vi termini), as to ratiocination, or explicit reasoning, and may therefore be accepted without error, and much to his profit, by
the logician. According to Horne Tooke "thing" and "think" are but the same word spelt differently; and hence, he says, "the vulgar pronunciation of 'nothink' instead of 'nothing' is not so very absurd."

L—§ 53

BOOLE'S LOGIC

"All the operations of language, as an instrument of reasoning, may," it is claimed by Mr. Boole, "be conducted by a system of signs composed of the following elements," viz.:

1st. "Literal symbols, as $x$, $y$, etc., representing names or terms.

2d. "Signs, as $+$, $-$, $\times$," representing relations to each other of the substantive elements of complex terms.

3d. "The sign of identity ($=$)," or, as I should call it, the sign of equivalence, i.e., of significative equivalence, or equivalence of denotation.

The names signified by signs of the first class may be either single names denoting classes,—as, e.g., man, horse, good, white, etc.,—or they may be composed of several names, denoting classes that partially coincide—as, e.g., good men, black sheep, etc. In the latter case the signs may be combined together precisely as the words denoting the terms. Thus, if we represent the class men by $x$, and the class good by $y$, "good men" will be denoted by the expression $yx$. So if $x$ stands for sheep, $y$ for black things, and $z$ for horned things, $zyx$ will denote "horned black sheep." But it is obvious that in the expression "black sheep," the
order in which the component terms are placed makes no difference; or, in other words, that it is the same thing whether we say "black sheep," as in English, or "sheep black," as in Spanish and other languages. Consequently, the class "black sheep" may be written either $yx$ or $xy$, which may be expressed by the following equation:

$$ (1) \ yx = xy. \quad \begin{array}{cc} x & Y \hline X & \end{array} $$

In which the complex term $yx$ or $xy$ denotes a class of individuals that is at once included in the class $y$ and the class $x$. On the same principle, if we represent by $z$ the adjective "horned," $zyx$ will stand for the term "horned black sheep," and we will have the following equations:

$$ (2) \ zxy = xyz = yxz. \quad \begin{array}{cc} x & \hline X & Y \hline Z & XYZ \end{array} $$

If, in the equation $xy = yx$, we suppose $y$ to be wholly included in $x$,—as, e. g., if it denote the black sheep in the flock $x$,—then we will have the equation:

$$ (3) \ xy = y. \quad \begin{array}{cc} y \hline \hline \end{array} $$

Again, if $x = y$, then $xy = x^2$. But a class is not
enlarged or diminished by repeating the term denoting it. Thus, "white white" or "sheep sheep" mean nothing more than "white" or "sheep." Hence we have the equation:

\[ (4) \quad x^2 = x. \]

If the class denoted by a term is composed of two classes, denoted respectively by \( x \) and \( y \), as, e.g., "men and women," it may be expressed by the complex term \( x + y \). But obviously, the expressions, "men and women," and "women and men," are equivalent in meaning. Hence the equation:

\[ (5) \quad x + y = y + x. \]

Again, if we qualify the term "men and women" by the adjective "Asiatic," we have the expression "Asiatic men and women"; but this is equivalent in meaning to the expression "Asiatic men and Asiatic women." Hence, denoting men by \( x \), women by \( y \), and Asiatic by \( z \), we have the equation:

\[ (6) \quad z(x + y) = zx + zy. \]

If we denote the adult population of a city by \( x \), and the women by \( y \), then \( x - y \) will denote the men. But it is indifferent whether we express the excepted class first or last, provided it be distinctly represented as the exception. Thus the expression, "the adult population less the women," and the expression, "excepting the women, the adult
population,“ are equivalent in meaning to each other, and both to the expression “the men.” Hence we have the equation:

\[(7) \ x - y = -y + x.\]

But the expression, “the white population, less the women,” is equivalent in meaning to the expression, “the white population, less the white women.”

Hence, representing “white” by \(z\), we have the equation:

\[(8) \ z (x - y) = zx - zy.\]

If, in the proposition, “The stars are the suns and the planets,” we denote stars by \(x\), suns by \(y\), and planets by \(z\), we shall have the equation:

\[(9) \ x = y + z.\]

But, if the stars are the suns and the planets, the stars, except the planets, are suns. Hence we have the equation:

\[(10) \ x - z = y.\]

If the terms \(x\) and \(y\) are equivalent, it is obvious that those of the class \(x\), or, as we may say, the \(x\)'s, that possess a given quality, must be identical with the \(y\)'s that possess it. Hence, if \(x = y\), we have the equation:

\[(11) \ zx = zy.\]
But, *per contra*, it cannot be inferred from the equation, \( zx = zy \), that \( x = y \). (§ 82 (2) n.)

For, "suppose it true that those members of a class \( x \) which possess a certain quality, \( z \), are identical with those members of a class \( y \), which possess the same quality, \( z \), it does not follow that the members of the class \( x \) universally are identical with the members of the class \( y \)." Thus, returning to our sheep, let \( x \) denote one portion of a flock of sheep, and \( y \) another, and let \( z \) denote "horned"; then \( zx \) will denote the horned sheep in one portion of the flock, and \( zy \) the horned sheep in the other; and, if we suppose these to be equal, we shall have the equation:

\[ zx = zy. \]

But it will not follow that the two portions of the flock are equal in number, and we therefore cannot say \( x = y \); as may be thus illustrated:

\[
\begin{array}{c}
\cap \\
\cup \\
\cap
\end{array}
\]

Adverting to the above equations, it will be perceived that the laws governing the convertibility of the different forms of expression are, to a certain extent, identical with those obtaining in mathematics. Thus, in the equations (1) and (2), the symbols are *commutative* like the symbols of algebra. The logical process here involved is, therefore, expressed in the same manner as in the correspond-
ing algebraic expression; and this expression, whether regarded as logical or algebraic, will be subject to the same law. There is, therefore, in the process involved in these equations, (1) and (2), a certain resemblance or analogy to the process of multiplication; and this is also true of equation (11).

In equations (6) and (8) a process is exhibited closely resembling that of factoring in algebra.

In equations (5), (7), (9), and (10), we have illustrated a principle of conversion of symbols apparently identical with the corresponding process in algebra. Hence we may affirm as logical axioms: 1st, that if equals be added to equals the wholes will be equal; and, 2d, that if equals be taken from equals, the remainders will be equals.

Hence, with regard to the equations specified (1, 2, 11, 6, 8, 5, 7, 9, and 10), we may affirm generally that the logical symbols may be transposed or converted precisely in the same way as in the operations of addition, subtraction, and multiplication in algebra. But with regard to the analogy between multiplication and the corresponding logical operation, it will be observed that in one respect it fails, namely, in equation (4), \( x^2 = x \); which is good in Logic, but not generally true in algebra. Also, it will be observed, there is apparently no logical process corresponding to the algebraic operation of division. Thus, as we have seen, we cannot infer from equation (11), "\( zx = zy \)," that \( x = y \), as we may in algebra.

But if we conceive of an algebra or arithmetic
that deals only with the two numbers, 1 and 0, this discrepancy will altogether disappear. For on such hypothesis, equation (4), \( x^2 = x \), will be true, both in Logic and in mathematics. And in equation (11), \( zx = zy \), if \( z = 1 \), the proposition, \( x = y \), may be inferred, both in Logic and in mathematics. But if \( z \) be equal to zero, it cannot be thus inferred, either in Logic or algebra. Hence, if we conceive of an algebra in which the symbols \( x \), \( y \), \( z \), etc., "admit indifferently of the values of 1 and 0 and of those values alone," then "the laws, the axioms, and the processes of such an algebra will be identical in their whole extent with the laws, axioms, and process of an algebra of Logic."

Accordingly, Mr. Boole's system is founded on this hypothesis, and "the logical value and significance" of the terms dealt with (1 and 0) are thus explained. In algebra, the equation \( oy = 0 \) is true, whatever the value of \( y \). So, in Logic, if \( o \) be regarded as a class, whatever class may be denoted by \( y \), the equation \( oy = 0 \) will be true; for, as we have seen, \( oy \) denotes the class of individuals that are at the same time included in the two classes,—\( i. e. \), 0 and \( y \). But none are included in the class \( o \), and therefore, \( oy = 0 \).

So in algebra, the equation \( iy = y \) is true, whatever the value of \( y \) may be, and this is true in Logic also, if \( i \) be regarded as including \( y \). For as we have seen (equation 3), if one of the two terms making a combined term is included in the other, the combined term is equal to the term of least extension. But this condition may be satisfied by regarding 1
as denoting the Universe. "Hence, the respective interpretations of the symbols, 0 and 1, in the system of Logic, are Nothing and Universe." Denoting the Universe by 1, and men by x, the expression 1 - x denotes the class "not-men," — i.e., all animals that are not men.

The equation \( x^2 = x \) may be put in the form, \( x^2 - x = 0 \), and this again in the form, \( x (1 - x) = 0 \); of which the interpretation is obvious; for, if \( x \) denotes "men," and \( 1 - x \) "not-men," it is clear that there can be no individuals belonging at once to the two classes, \( x \) and \( 1 - x \), or, men and not-men. So if we denote by \( x \) any class characterized by the possession of any quality whatever the same result will follow.

It is observed by Mr. Boole that the principle of analysis and classification involved in his system is "division into pairs of opposites, or, as it is technically said, Dichotomy" (§ 47), and this is in fact the fundamental process in Logic. And this, it will be observed, agrees with the opinion of Hobbes and of Aristotle (§ 90 n.).

In equation (5), it will be observed, there is a certain ambiguity in the expression \( x + y \). In common speech the classes denoted by the symbols \( x \) and \( y \) may either be exclusive of each other, or they may overlap, as, for instance, in the proposition, "Scholars and men of the world desire happiness," or, "Useful things are those that either produce pleasure, or prevent pain." In Mr. Boole's system this ambiguity is removed.

If the two classes are intended to include each
other, the expression to denote the aggregate class will be \( x(1-y) + y(1-x) \); which is to be read x’s that are not y’s; and y’s that are not x’s.

If we intend two classes that overlap, then the full expression should be, \( xy + x(1-y) + y(1-x) \).

"The result of these investigations may be embodied in the following rule of expression:

"**Rule.**—Express simple names or qualities by the symbols \( x, y, z, \) etc., their contraries by \( 1-x, 1-z, \) etc.; classes of things defined by common names or qualities, by connecting the corresponding symbols as in multiplication; collections of things consisting of portions different from each other, by connecting the expressions of those portions by the sign +. In particular, let the expression, ‘Either x’s or y’s’ be expressed by \( x(1-y) + y(1-x) \) when the classes denoted by \( x \) and \( y \) are exclusive; by \( x + y (1-x) \) when they are not exclusive. Similarly let the expression, ‘Either x’s or y’s or z’s’ be expressed by \( x(1-y)(1-z) + y(1-x)(1-z) + z(1-x)(1-y) \), when the classes denoted by \( x, y, \) and \( z \) are designed to be mutually exclusive; and by \( x + y (1-z) + z (1-x) (1-y) \), when they are not meant to be exclusive, and so on."

For illustration, "let us assume

\[
x = \text{hard}, \quad y = \text{elastic}, \quad z = \text{metals};
\]

and we shall have the following results:

"‘Non-elastic metals’ will be expressed by \( z(1-y) \);"
"‘Elastic substances with non-elastic metals’ by 
\[ y + z (1 - y); \]
"‘Hard substances, except metals,’ by \[ x - y; \]
"‘Metallic substances, except those which are 
neither hard nor elastic,’ by \[ z - z (1 - x) (1 - y), \]
or by \[ z [1 - (1 - x) (1 - y)]. \]
"

The above brief account of the elements of Mr. Boole’s system is given for the purpose of illustrating the laws that govern the convertibility of terms, and of substantive elements of terms; or, in other words, that govern the formal substitution of equivalent expressions, (§ 67 (2))—a purpose for which it admirably serves. It will require some attention to understand it, but with such attention, no difficulty will present itself.

It may be readily perceived that by the use of the above data a very extensive calculus may be developed, and such a one has in fact been developed by Mr. Boole; but with regard to its utility, opinions may widely differ.

"The idea of a logical calculus," says Lotze, "has been often taken up and often abandoned; but the Englishman Boole has recently made an elaborate and careful attempt to carry it out, which is beginning to attract attention in Germany, as well as in his own country. Though I freely admit that the author’s ingenuity makes his able work very charming, I am unable to convince myself that this calculus will help us to solve problems which defy the ordinary methods of Logic.”

(Logic, vol. ii., 277.)
M—§ 96

TABLE OF SYLLOGISMS

\[ \text{1st Figure} \left\{ \begin{array}{c} VX \\ ZY \\ ZX \end{array} \right. \]

\textit{Barbara}
A: Y is X
A: Z is Y
A: \therefore Z is X

\textit{Celarent}
E: Y is not X
A: Z is Y
E: \therefore Z is not X

\textit{Darii}
A: Y is X
I: Some Z is Y
I: \therefore Some Z is X

\textit{Ferio}
E: Y is not X
I: Some Z is Y
O: \therefore Some Z is not X

\textit{Cesare}
E: X is not Y
A: Z is Y
E: \therefore Z is not X

\textit{Celarent}
E: Y is not X
A: Z is Y
E: \therefore Z is not X
Camestres
A: X is Y
E: Z is not Y
E: \( \therefore \) Z is not X

Festino
E: X is not Y
I: Some Z is Y
O: \( \therefore \) Some Z is not X

Fakoro
A: X is Y
O: Some Z is not Y
O: \( \therefore \) Some Z is not X

Ferio
E: Not-Y is not X
I: Some Z is not Y
O: \( \therefore \) Some Z is not X

Darapti
A: Y is X
A: Y is Z
I: \( \therefore \) Some Z is X

Disamis
I: Some Y is X
A: Y is Z
I: \( \therefore \) Some Z is X

Celarent
E: Y is not Z
A: X is Y
E: \( \therefore \) X is not Z or Z is not X

Darii
A: Y is Z
I: Some X is Y
I: \( \therefore \) Some X is Z or Some Z is X
Appendix Notes

Datisi
A: Y is X
I: Some Y is Z
I: ′. Some Z is X

Felapton
E: Y is not X
A: Y is Z
O: ′. Some Z is not X

Dokamo
O: Some Y is not X
A: Y is Z
O: ′. Some Z is not X

Ferison
E: Y is not X
I: Some Y is Z
O: ′. Some Z is not X

Darrii
A: Y is Z
I: Some not X is Y
I: ′. Some not X is Z
or, Some Z is not X

Ferio
E: Y is not X
I: Some Z is Y
O: ′. Some Z is not X

4th Figure


Bramantip
A: X is Y
A: Y is Z
I: ′. Some Z is X

Camenes
A: X is Y
E: Y is not Z
O: ′. Z is not X

Barbara
A: Y is Z
A: X is Y
A: ′. X is Z
or Some Z is X

Celarent
E: Y is not Z
A: X is Y
E: ′. X is not Z
or Z is not X
The opinion of Locke cited, which occurs at the end of his essay, may be taken as the consummation and final generalization of his theory of knowledge. In the body of the work the conclusion reached by him is, that the elements of all knowledge are ideas (by which is meant what are now commonly called notions or concepts), and that "knowledge [is] but the perception of the connection and agreement, or disagreement, or repugnancy of any of our ideas" (Essay, b. 4, c. 1).

This definition, it will be observed, is too narrow, as it excludes the knowledge derived directly from the perception of concrete objects. But allowing for this defect it is accurate and profound and must be taken as the foundation of all science. In the beginning it seems that Locke had no
conception, or at least a very inadequate conception of the intimate connection between language and thought, and of the indispensability of the former as an instrument of thought. But as he proceeded he seems gradually to have realized this great truth, — which is treated of in his third book; and upon the conclusions thus reached is based his theory of knowledge and his general philosophy as developed in his fourth book, and as generalized in the concluding chapter, to which we have referred. His theory of knowledge, therefore, is to be regarded as based to a great extent expressly, and otherwise implicitly, upon the notion that all knowledge beyond that coming from experience consists in the perception of the agreement, or disagreement, of our ideas, or notions; and hence that all reasoning must consist in the comparison of notions or concepts; that practically this can be effected only by means of the names of the concepts or notions; and hence that Logic must consist in Analysis and Synthesis of names or terms; which is the theory of this work. (See observation of Horne Tooke, Appendix A.)
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