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Jointly Adaptive Equalization and Carrier Recovery in Two-Dimensional Digital Communication Systems

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In this paper, we describe a novel receiver structure for two-dimensional-modulated, suppressed-carrier data signals. The receiver consists of a passband equalizer followed by a demodulator which compensates for frequency offset and phase jitter; the demodulator's phase angle is provided by a data-directed, carrier recovery loop, which is shown by analysis and simulation to be capable of tracking relatively high frequency phase jitter. A derivation of the receiver parameters is presented, based on a gradient algorithm for jointly optimizing the equalizer tap coefficients and the carrier phase estimate, to minimize the output mean-squared error. System performance is related to carrier phase-tracking parameters by analysis. Computer simulations confirm the feasibility of the receiver structure.

I. INTRODUCTION

In recent years, a number of double-sideband suppressed-carrier linear-modulation techniques have seen increasing application to the efficient transmission of digital data over band-limited channels. Two-dimensional modulation may be an appropriate designation for these techniques, since they call for coding the transmitted data as two-dimensional data symbols and transmitting the two components by amplitude-modulating two quadrature carrier waves.

Phase-shift keying (PSK) and quadrature amplitude modulation (QAM, sometimes termed QASK), illustrated in Fig. 1, are familar examples. Other two-dimensional modulation examples, characterized by their signal constellations (discrete sets of two-dimensional data symbols), have been extensively studied.[1–3]

This paper presents a unified treatment of adaptive equalization, carrier recovery, and demodulation for two-dimensional-modulated data communication systems. Most previous studies of QAM and PSK systems have treated these receiver functions separately.[4–8] Kobayashi

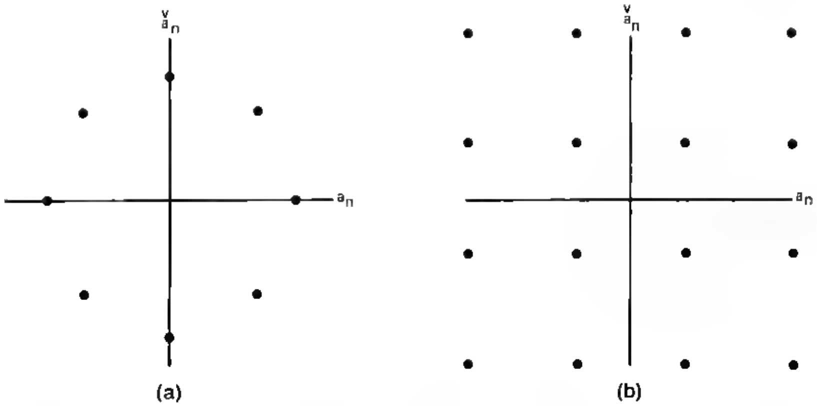

Fig. 1—Examples of two-dimensional signal constellations. (a) 8-phase PSK. (b) 16-point QAM.

presented a unified receiver structure applicable to two-dimensional modulation, based on maximum-likelihood reception.[9] Chang studied a unified linear receiver structure for (one-dimensional) single-sideband modulation systems.[10] A novel feature of the receiver structure presented in this paper is the placement of the carrier phase-tracking and demodulation functions together, after adaptive passband equalization.† In a more traditional receiver arrangement,[8–10] baseband equalization follows demodulation and precedes decision-directed phase estimation, thereby introducing a delay of many symbol intervals between these two functions. The decision-directed phase estimate is therefore a *delayed* version of the true channel phase shift affecting the signal that is entering the demodulator. This delay would lead to inaccurate demodulation of a signal perturbed by a time-varying phase shift (phase jitter) introduced by some channels. The receiver structure presented here avoids this source of inaccuracy by placing both the demodulation and phase estimation functions *after* the equalizer.

In Section II we introduce complex notation for two-dimensional bandpass signals and for the effects of linear distortion, phase jitter, and frequency offset. Section III introduces the receiver structure and reviews the function of the passband equalizer. Section IV introduces a mean-squared-error criterion and proposes a gradient algorithm for arriving at a (nonunique) set of equalizer tap coefficients and a carrier phase estimate to minimize it. This ideal gradient algorithm is the motivation for a joint decision-directed equalizer up-

† The receiver structure and an equivalent implementation of it are depicted in Figs. 2 and 3, respectively.

dating and demodulation phase-tracking algorithm. It is shown that the phase-tracking algorithm performs essentially the function of a first-order phase-locked loop operating in discrete time. A very simple linear analysis of the loop in Section V illustrates its phase-jitter tracking capability. The receiver's capabilities are further confirmed by the results of simulations, reported in Section VI.

II. BANDPASS SIGNALS AND PHASE JITTER

We consider double-sideband, suppressed-carrier, two-dimensional-modulated data signals specified by

$$s(t) = \sum_n a_n g(t - nT) \cos 2\pi f_c t - \sum_n \check{a}_n g(t - nT) \sin 2\pi f_c t, \quad (1)$$

where f_c is the carrier frequency, $g(t)$ is a suitably chosen baseband pulse waveform, T is the duration of a symbol interval, and the pair $(a_n, \check{a}_n)$ represents a discrete-valued two-dimensional data symbol. For example, in a 16-point QAM system, each a_n and $\check{a}_n$ is chosen independently from the set $\{\pm 1, \pm 3\}$. In a phase-modulation system (PSK), a_n and $\check{a}_n$ have the form $a_n = \cos \psi_n$ and $\check{a}_n = \sin \psi_n$, the information being coded onto the phase ψ_n. These examples are displayed in Fig. 1.

It is convenient to deal only with the *positive* frequency content of passband spectra. The associated time functions are complex-valued. Thus $s(t) = \text{Re}\,[s(t) + j\check{s}(t)]$, where $\check{s}(t)$ is the Hilbert transform of $s(t)$ and $[s(t) + j\check{s}(t)]$ possesses a Fourier transform consisting of twice the positive frequency part of the spectrum of $s(t)$:

$$s(t) + j\check{s}(t) = \sum_n A_n g(t - nT) \exp(j2\pi f_c t), \quad (2)$$

where $A_n = a_n + j\check{a}_n$. The complex passband waveform $g(t - nT) \times \exp(j2\pi f_c t)$ is said to be *analytic* if its spectrum is nonzero only for positive frequencies. In general, we shall represent real quantities by lower-case letters and complex ones by upper-case letters.

When $s(t)$ is passed through a noisy linear channel, the output is expressed as

$$s'(t) \equiv \text{Re}\,\{\sum_n A_n C(t - nT) \exp[j(2\pi f_c t + \theta)]\} + n(t), \quad (3a)$$

where $C(t)$ is a complex baseband equivalent impulse response of the combined transmitting filter and channel, θ a phase shift that may be inserted by the channel, and $n(t)$ a realization of additive noise.

Some channels introduce a time-varying phase shift, expressed in general as

$$\theta(t) \equiv \theta + 2\pi\Delta t + \psi(t).$$

Here θ represents a fixed phase shift, Δ a fixed frequency offset, and $\psi(t)$ a random or quasi-periodic waveform that is a manifestation of pbase jitter. On voiceband telepbone channels, tbe peak magnitude of tbe waveform $\psi(t)$ is usually less tban about 10 degrees, and its highest frequency spectral component is typically less than 10 percent of the data signal's bandwidth.[11] If tbe typical rate of variation were comparable to tbe symhol rate $1/T$, a mathematical model for phase jitter would be critically dependent on tbe linear filter transfer functions preceding and following the location wbere tbe channel phase-modulates tbe data signal with the phase jitter. However, the assumption of small, relatively slow phase jitter permits us to sidestep this distinction and to model the phase-jitter-perturbed received signal conveniently as

$$s'(t) \equiv \mathrm{Re}\,\{\sum_{n} A_n C(t - nT)\exp[j(2\pi f_c t + \theta_n)]\} + n(t); \quad \text{(3b)}$$

i.e., θ_n is interpreted as the channel phase shift affecting the transmission of the nth data symbol A_n.

III. RECEIVER STRUCTURE

Figure 2 shows the two-dimensional receiver structure. Tbe real-valued received waveform $s'(t)$ first enters a phase splitter, consisting of parallel passband filters with impulse responses $h(t)$ and $\check{h}(t)$, where $\check{h}(t)$ is the Hilbert transform of $h(t)$; thus the complex impulse response defined hy $H(t) \equiv h(t) + j\check{h}(t)$ is analytic. An appropriate choice for $H(t)$ is a filter matched to the transmitted pulse, i.e.,

$$H(t) = g(-t)\exp(j2\pi f_c t). \quad (4)$$

If the channel $C(t)$ were known *a priori*, an optimal choice for $H(t)$ would be a matched filter impulse response

$$C(-t)^* \exp(j2\pi f_c t).$$

The optimality of the complex matched filter and sampler for two-dimensional modulation is brought out in the studies of Kobayashi,[9]

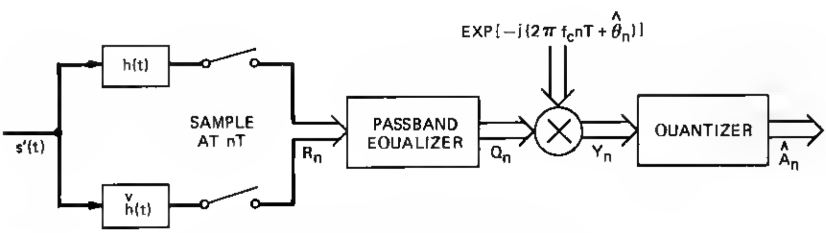

Fig. 2—Two-dimensional receiver.

Ungerboeck,[12] and Ericson and Johansson.[13] Sampling is carried out at the symbol rate $1/T$. We assume a fixed choice of sampling phase and will not be concerned with its optimization. The problem of deriving the optimum sampling phase has been treated previously.[14–16]

The pair of outputs at time nT from the sampler r_n and $\check{r}_n$ can be expressed as a complex sample

$$R_n \equiv r_n + j\check{r}_n,$$

wbich is of the form

$$R_n = \sum_k A_k X_{n-k} \exp[j(2\pi f_c nT + \theta_k)] + N_n, \tag{5}$$

where $X_n \equiv X(nT)$ is a sample of tbe overall complex baseband equivalent impulse response and N_n is a complex sample of filtered noise.

The passband linear equalizer[7] with, say, $2M + 1$ complex tap coefficients $\{C_n^*\}_{-M}^{M}$ produces complex passband output samples $\{Q_n\}$ which are a linear combination of sampled inputs; i.e.,

$$Q_n \equiv q_n + j\check{q}_n = \sum_{k=-M}^{M} C_k^* R_{n-k}. \tag{6a}$$

Note that the equalizer's implementation is described either by the above complex expression or by two expressions for the two real outputs, viz.,

$$q_n = \sum_{k=-m}^{M} (c_k r_{n-k} + \check{c}_k \check{r}_{n-k}) \tag{6b}$$

$$\check{q}_n = \sum_{k=-M}^{M} (c_k \check{r}_{n-k} - \check{c}_k r_{n-k}), \tag{6c}$$

where

$$C_k^* \equiv c_k - j\check{c}_k.$$

Expression (6) can also be expressed in vector notation. Define

$$\mathbf{C} \equiv \begin{pmatrix} C_M \\ \vdots \\ C_{-M} \end{pmatrix} \tag{7a}$$

$$\mathbf{R}_n = \begin{pmatrix} R_{n-M} \\ \vdots \\ R_{n+M} \end{pmatrix}. \tag{7b}$$

Then

$$Q_n = \mathbf{C}^*\mathbf{R}_n, \tag{8}$$

where * means complex conjugate transpose.

Ideally, the passband equalizer's sampled impulse response $\{C_k^*\}$ should be such as to yield an overall passband channel with no intersymbol interference; i.e.,

$$\text{ideal } Q_n \equiv A_n \exp[j(2\pi f_c nT + \theta_n)].$$

The information symbol A_n is then recovered by demodulating Q_n to baseband and quantizing the result in accordance with the two-dimensional signal constellation. If the demodulator has a phase estimate $\hat{\theta}_n$, the complex demodulated output is given by

$$Y_n \equiv y_n + j\check{y}_n = Q_n \exp[-j(2\pi f_c nT + \hat{\theta}_n)] \tag{9a}$$

or

$$y_n = q_n \cos(2\pi f_c nT + \hat{\theta}_n) + \check{q}_n \sin(2\pi f_c nT + \hat{\theta}_n) \tag{9b}$$

$$\check{y}_n = -q_n \sin(2\pi f_c nT + \hat{\theta}_n) + \check{q}_n \cos(2\pi f_c nT + \hat{\theta}_n). \tag{9c}$$

The ideal output at time nT is A_n and the receiver *error* is defined by

$$E_n \equiv Y_n - A_n. \tag{10a}$$

For the joint optimization of the equalizer tap coefficients and the demodulator phase, we adopt the following mean-squared-error criterion: minimize ϵ_n, where

$$\epsilon_n \equiv \langle |E_n|^2 \rangle \tag{10b}$$

and the expectation, denoted by $\langle\ \rangle$, is over the data sequence and noise.†

The receiver structure shown in Fig. 2 is characterized by the following expression for the complex output sample before quantization. From (6a) and (9a),

$$Y_n = \left[\sum_{k=-M}^{M} C_k^* R_{n-k}\right] \exp[-j(2\pi f_c nT + \hat{\theta}_n)]. \tag{11}$$

An alternative equivalent receiver has a "hasehand" structure. Define a new set of tap coefficients by

$$C_k^{*\prime} \equiv C_k^* \exp(-j2\pi f_c kT) \tag{12a}$$

and a set of demodulated received samples by

$$R_n' \equiv R_n \exp(-j2\pi f_c nT). \tag{12b}$$

† The "symmetric" mean-squared-error criterion (10b) was proposed by R. D. Gitlin and K. H. Mueller as an improvement to the "single-sided" criterion proposed in Ref. 7.

Then (11) can be re-expressed as

$$Y_n = \left[\sum_{k=-M}^{M} C_k^{*\prime} R'_{n-k} \right] \exp(-j\hat{\theta}_n). \tag{12c}$$

The fully equivalent implementation expressed by (12c) is depicted in Fig. 3. Note that the received samples are demodulated to baseband using a free-running oscillator as in (12b) *before* equalization. However, a second stage of demodulation following baseband equalization remains, whose purpose is to remove the effects of channel phase variation. Again, the delay of the equalizer does not come between this secondary demodulation and the derivation of the phase estimate $\hat{\theta}_n$. The equivalence of the "passband" and "baseband" receiver implementations of Figs. 2 and 3, respectively, gives the system designer some extra flexibility.

IV. OPTIMIZATION OF EQUALIZER TAP COEFFICIENTS AND DEMODULATION PHASE

To bring out the relationships governing the optimal tap vector $\mathbf{C}_n$ and demodulator phase $\hat{\theta}_n$ (both of which may be functions of time), we assume that successive data symbols are uncorrelated; i.e.,

$$\begin{aligned} \langle A_l A_m \rangle &= 0 \qquad \text{all} \quad l, m \\ \langle A_l A_m^* \rangle &= \langle |A|^2 \rangle \delta_{lm}, \end{aligned} \tag{13}$$

where δ_{lm} is the Kronecker delta function. Then for future reference we note, from expressions (5) and (7b), that cross correlation of the data symbols with sampled phase-splitter outputs results in

$$\langle A_n^* \mathbf{R}_n \rangle = \mathbf{X}_n \exp[j(2\pi f_c nT + \theta_n)] \langle |A|^2 \rangle, \tag{14}$$

where

$$\mathbf{X} \equiv \begin{pmatrix} X_{-M} \exp(-j2\pi f_c MT) \\ \vdots \\ X_M \exp(j2\pi f_c MT) \end{pmatrix} \tag{15}$$

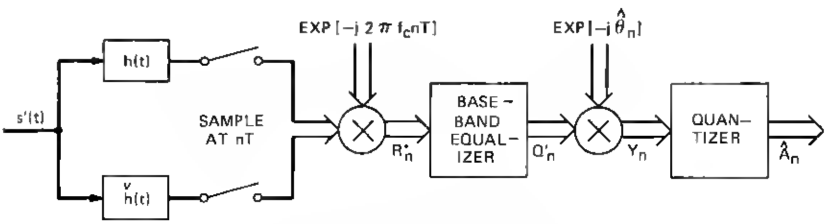

Fig. 3—Equivalent implementation.

is the complex impulse response vector of the combination of the transmitter pulse filter and the channel, truncated to $2M + 1$ samples. The channel correlation matrix or A matrix is defined to be

$$A \equiv \frac{\langle \mathbf{R}_n \mathbf{R}_n^* \rangle}{\langle |A|^2 \rangle}. \tag{16}$$

This is a Hermitian matrix ($A^* = A$) whose l-mth element is

$$A_{lm} = \sum_n X_n X_{n+m-l}^* \exp[j2\pi f_c (l - m)T] + \rho_{l-m}, \tag{17}$$

where $\{\rho_{l-m}\}$ is the noise autocorrelation. Furthermore, it is positive semidefinite. (For any vector $\mathbf{u}$, $\mathbf{u}^* A \mathbf{u} = \langle |\mathbf{u}^* \mathbf{R}_n|^2 \rangle \geqq 0$.)

Using definitions (10), (11), (14), and (16), we can rewrite ϵ_n in terms of A and $\mathbf{X}$, which are fundamental characteristics of the channel.

$$\epsilon_n = \{\mathbf{C}_n - A^{-1}\mathbf{X} \exp[-j(\hat{\theta}_n - \theta_n)]\}^* \cdot A\{\mathbf{C}_n - A^{-1}\mathbf{X} \exp[-j(\hat{\theta}_n - \theta_n)]\} + 1 - \mathbf{X}^* A^{-1} \mathbf{X}. \tag{18}$$

Because the matrix A is positive semidefinite, ϵ_n has the unique minimum

$$\epsilon_{\min} = 1 - \mathbf{X}^* A^{-1} \mathbf{X}, \tag{19}$$

which is achieved when $\mathbf{C}_n$ and $\hat{\theta}_n$ satisfy

$$\mathbf{C}_n = \mathbf{C}_{n\,\mathrm{opt}}(\hat{\theta}_n) \equiv A^{-1}\mathbf{X} \exp[-j(\hat{\theta}_n - \theta_n)]. \tag{20}$$

Observe that the solution (20) is not unique; there is an infinitude of combinations ($\mathbf{C}_n$, $\hat{\theta}_n - \theta_n$) that yield the minimum. However, for any specific choice of $\hat{\theta}_n$ (including zero), there is a unique optimum choice of $\mathbf{C}_n$. Indeed, this is a manifestation of the "tap-rotation" property of the passband equalizer which was pointed out by Gitlin, Ho, and Mazo.[7] In particular, when there is no attempt to estimate $\theta_n (\hat{\theta}_n = 0)$, then any amount of frequency offset $\Delta (\theta_n = 2\pi n \Delta T)$ causes $\mathbf{C}_{n\,\mathrm{opt}}$ to "rotate" with frequency Δ. However, a typical adaptive equalizer whose tap coefficients may not be permitted to change by more than about 1 percent from one symbol interval to the next will not be able simultaneously to equalize the channel effectively and to rotate $2\pi\Delta$ radians per symbol interval even for moderate amounts of frequency offset. Similarly, the equalizer taps could not be expected to track typical phase jitter components accurately.

The principal innovation reported in this paper is the joint operation of the adaptive equalizer and a separate phase-tracking loop which removes the major burden of tracking from the slowly adapting equalizer. Assuming this separate phase-angle-tracking algorithm is successful so that the phase error ($\hat{\theta}_n - \theta_n$) remains constant, we observe, by writing the mean-squared error using definitions (10) and (11) as

$$\epsilon_n \equiv \frac{1}{\langle |A|^2 \rangle} \langle |\mathbf{C}_n^* R_n - A_n \exp[j(2\pi f_c nT + \hat{\theta}_n)]|^2 \rangle, \qquad (21)$$

that, if the equalizer's reference signal for the purpose of adapting its tap coefficients is $\{A_n \exp[j(2\pi f_c nT + \hat{\theta}_n)]\}$, the reference signal rotates *in synchronism* with the frequency-offset and phase-jittered carrier of the received signal, and hence the equalizer tap coefficients do not have to rotate if $\theta_n - \hat{\theta}_n$ remains constant.

If the gradients of ϵ_n with respect to the real tap coefficient vectors $\mathbf{c}_n$ and $\check{\mathbf{c}}_n$ are denoted respectively by $\nabla_{\mathbf{c}_n}\epsilon_n$ and $\nabla_{\check{\mathbf{c}}_n}\epsilon_n$ and if we define the gradient with respect to $\mathbf{C}_n$ to be

$$\nabla_{\mathbf{C}_n}\epsilon_n \equiv \nabla_{\mathbf{c}_n}\epsilon_n + j\nabla_{\check{\mathbf{c}}_n}\epsilon_n,$$

then the gradient of the right-hand side of (18) can be written

$$\nabla_{\mathbf{C}_n}\epsilon_n = 2\{A\mathbf{C}_n - \mathbf{X}\exp[-j(\hat{\theta}_n - \theta_n)]\}. \qquad (22)$$

Then it follows from expression (18) and from the fact that A is positive semidefinite that $\nabla_{\mathbf{C}_n}\epsilon_n = 0$ is a necessary and sufficient condition for ϵ_n to attain its minimum value.* If the receiver knew $\mathbf{X}\exp(j\theta_n)$, defined by (15), and A, defined by (16), and could calculate this gradient during each symbol interval, then in the nth symbol interval it could use a gradient algorithm to update its estimate of $\mathbf{C}_n$ as follows:

$$\mathbf{C}_{n+1} = \mathbf{C}_n - \frac{\beta}{2}\nabla_{\mathbf{C}_n}\epsilon_n, \qquad (23)$$

where the gradient is defined by (22). In this equation, $\mathbf{C}_n$ is the estimate of the correct tap coefficient vector in the nth symbol interval and $\beta/2$ is a positive constant. For the moment, we defer consideration of a more realistic algorithm that does not require prior knowledge of A and $\mathbf{X}$.

Let us now consider the means for providing the estimated sequence $\{\hat{\theta}_n\}$. In general, of course, the true phase jitter angle sequence $\{\theta_n\}$ is a random process. However, the reasonable assumption that it varies slowly with n leads us to treat θ_n as a quasi-static parameter that must be estimated in symbol interval n from present and past received data $\{R_n\}$ and reference information symbols $\{A_n\}$.

Accordingly, the receiver will incorporate an algorithm for updating its estimate $\hat{\theta}_n$, based on a gradient search technique. The derivative

* We assume that matrix A is nonsingular.

of ϵ_n with respect to $\hat{\theta}_n$ is, for a fixed value of $\mathbf{C}_n$,

$$\nabla_{\hat{\theta}_n}\epsilon_n = -2\,\mathrm{Im}\,\{\mathbf{C}_n^*\mathbf{X}\exp[-j(\hat{\theta}_n - \theta_n)]\}. \tag{24}$$

The estimate $\hat{\theta}_n$ is thus updated as

$$\hat{\theta}_{n+1} = \hat{\theta}_n - \frac{\alpha}{2}\nabla_{\hat{\theta}_n}\epsilon_n, \tag{25}$$

where $\alpha/2$ is a constant. In general, α should be large relative to the equalizer's constant β, to ensure that the estimate $\hat{\theta}_n$ can closely track a varying angle θ_n, therehy ohviating the need for the passband equalizer taps to follow it closely.

Suppose the angle θ_n is not time-varying ($\theta_n = \theta$). Then the stationary points of the gradient algorithms (23) and (25) are the solutions of the equations

$$\nabla_{\mathbf{C}}\epsilon_n = 0$$

or

$$A\mathbf{C} = \mathbf{X}\exp[-j(\hat{\theta} - \theta)], \tag{26}$$

and

$$\nabla_{\hat{\theta}}\epsilon_n = 0$$

or

$$\mathrm{Im}\,\{\mathbf{C}^*\mathbf{X}\exp[-j(\hat{\theta} - \theta)]\} = 0. \tag{27}$$

It is easy to show from the Hermitian property of A that, if (26) is true, then (27) is true. Furthermore, A is positive semidefinite and thus expression (18) for the mean-squared error shows that the infinite set of stationary points, defined by (26), are the only global minima.

The following question immediately arises: Starting with fixed initial values, $\mathbf{C}_0$ and $\hat{\theta}_0$ and assuming $\theta_n = \theta$ for all n, do the gradient algorithms (23) and (25) jointly converge to a stationary point? Note that by defining

$$\mathbf{Z}_n = \begin{pmatrix} \mathbf{C}_n \\ \hline \hat{\theta}_n \end{pmatrix}, \qquad P = \begin{bmatrix} \frac{\beta}{2} & 0 \\ 0 & \frac{\alpha}{2} \end{bmatrix}$$

and

$$\nabla_{\mathbf{Z}_n}\epsilon_n = \begin{pmatrix} \nabla_{\mathbf{C}_n}\epsilon_n \\ \hline \nabla_{\hat{\theta}_n}\epsilon_n \end{pmatrix},$$

we can combine (23) and (25) by writing

$$\mathbf{Z}_{n+1} = \mathbf{Z}_n - P\nabla_{\mathbf{Z}_n}\epsilon_n. \tag{28}$$

It is shown in Ref. 17 that, if β and α are chosen small enough, the sequence $\{\mathbf{Z}_n\}$ converges in mean-square to a stationary point for which $\nabla_Z \epsilon_n = 0$. As pointed out previously, the stationary points all yield the global minimum of the mean-squared error and thus (28) converges to yield the minimum mean-squared error. The question of rate of convergence will be treated in a later paper.

In a practical situation, the receiver does not know *a priori* the ensemble averages represented by the channel correlation matrix A and the truncated impulse response vector $\mathbf{X}$. In this situation, the receiver can approximate the gradient search algorithm by utilizing the gradients with respect to $\mathbf{C}$ and $\hat{\theta}_n$ of the actual unnormalized squared error

$$|E_n|^2 = |\mathbf{C}_n^* \mathbf{R}_n - A_n \exp[j(2\pi f_c nT + \hat{\theta}_n)]|^2$$

instead of its mean. The A_n used in this calculation is initially an ideal reference known to the receiver, and during normal operation it is the receiver's output decision $\hat{A}_n$ in the nth interval. Thus a decision-directed stochastic approximation algorithm corresponding to (23) and (25) is

$$\begin{aligned} \mathbf{C}_{n+1} &= \mathbf{C}_n - \frac{\beta}{\langle |A|^2 \rangle} \{\mathbf{R}_n \mathbf{R}_n^* \mathbf{C}_n - \hat{A}_n^* R_n \exp[-j(2\pi f_c nT + \hat{\theta}_n)]\} \\ &= \mathbf{C}_n - \frac{\beta}{\langle |A|^2 \rangle} \mathbf{R}_n (Q_n^* - \hat{Q}_n^*), \end{aligned} \tag{29}$$

where $\hat{Q}_n = \hat{A}_n \exp[j(2\pi f_c nT + \hat{\theta}_n)]$ is the "rotated" reference for the equalizer in the nth interval, using the receiver's decision $\hat{A}_n$, and

$$\begin{aligned} \hat{\theta}_{n+1} &= \hat{\theta}_n + \frac{\alpha \operatorname{Im}}{|A_n|^2} \{\mathbf{C}_n^* \mathbf{R}_n A_n^* \exp[-j(2\pi f_c nT + \hat{\theta}_n)]\} \\ &= \hat{\theta}_n + \frac{\alpha}{|A_n|^2} \operatorname{Im} (Q_n \hat{Q}_n^*), \end{aligned} \tag{30}$$

which can also be written as $\hat{\theta}_{n+1} = \hat{\theta}_n + \alpha/|A_n|^2 \operatorname{Im} \{Y_n \hat{A}_n^*\}$.

Expression (30) has a simple heuristic interpretation. Suppose the equalizer has successfully removed all intersymbol interference so that its output, neglecting noise, can be written

$$Q_n \approx A_n \exp[j(2\pi f_c nT + \theta_n)].$$

Then we can write (30) as

$$\hat{\theta}_{n+1} \approx \hat{\theta}_n - \alpha \sin(\hat{\theta}_n - \theta_n). \tag{31}$$

Equation (31) describes a discrete-time, first-order, phase-locked loop. Because the tracking algorithm makes use of the receiver's decisions, it can be termed a decision-directed tracking loop or a decision feedback loop.[5,6] As expressed in (31), the demodulator phase $\hat{\theta}_n$ is corrected by an amount proportional to the sine of the angular difference between the demodulated output Y_n and the receiver's decision $\hat{A}_n$. The maximum handwidth of the phase jitter that can he compensated for is a function of the constants α and β. This will be explored in the next section and in a suhsequent paper.

The decision-directed phase-tracking principle is well-known for application in systems that do not require adaptive equalization.[5,6] Its appropriateness is further confirmed hy studies of maximum-likelihood detection.[9,12]

Note that there is a phase amhiguity in the receiver's decisions inherent in suppressed-carrier systems with symmetric signal constellations, using decision-directed phase tracking. For example, the QAM signal constellation of Fig. 1 is quadrantally symmetric, and therefore constant 90-degree errors in the phase of the receiver's decisions $\{\hat{A}_n\}$ are undetectahle. This source of amhiguity is customarily removed hy differentially encoding the transmitted data onto the points of the signal constellation, so that phase *differences* between successive decisions $\{\hat{A}_n\}$, rather than absolute phase values, convey information.

V. THE PHASE-TRACKING GAIN CONSTANT α: TRACKING BANDWIDTH CONSIDERATIONS

As pointed out in Section IV, the phase-angle-tracking algorithm is, assuming perfect equalization, hasically that of a first-order phase-lock loop with gain constant α. The actual system does not hehave quite as simply as this, however, since the passhand equalizer, even with a small gain constant β, will also attempt to track the phase to some extent; i.e., the difference equations (29) and (30) are coupled. This coupling and its effect on performance will he explored in a later paper. In this section, we ignore this effect and also the effect of imperfect equalization. Furthermore, in view of the difficulty in analyzing discrete-time phase-locked systems, we make the following linearizing approximation for the steady-state phase error: $|\hat{\theta}_n - \theta_n| \ll \pi$, so that $\sin(\hat{\theta}_n - \theta_n) \approx \hat{\theta}_n - \theta_n$. We can write (31) as the simple linear difference equation

$$\hat{\theta}_{n+1} = (1-\alpha)\hat{\theta}_n + \alpha\theta_n. \tag{32}$$

The case of sinusoidal phase jitter, $\theta_n \equiv \text{Re}\,[J\exp(j\omega nT)]$, is of interest hecause the phase jitter ohserved on telephone channels often consists of one or more sinusoids with frequencies $\omega/2\pi$ Hz, which are harmonics of various power line frequencies. The response $\hat{\theta}_n = \text{Re}\,[\hat{J}\exp(j\omega nT)]$ of the linearized phase-locked loop to θ_n is easily found to be given hy

$$\frac{\hat{J}}{J} = F(j\omega) = \frac{\alpha}{\exp(j\omega T) - 1 + \alpha}. \tag{33}$$

Now let us consider the effect of additive noise on the linearized phase-tracking algorithm. Assume the complex, equalized, demodulated output can be written

$$Y_n = A_n \exp[-j(\hat{\theta}_n - \theta_n)] + V_n, \tag{34}$$

where $V_n = v_n + j\check{v}_n$ is a complex gaussian random variable with zero mean. Although in general successive noise samples at the equalizer output will be correlated, we assume they are uncorrelated to simplify the results. The effect of this simplification should be minor if the phase-tracking bandwidth is much smaller than the data bandwidth or if the frequency response of the channel and of the equalizer are both nearly flat. Thus if the signal-to-noise ratio is $\langle |A|^2 \rangle / N_0$, $\langle v_n v_m \rangle = \langle \check{v}_n \check{v}_m \rangle = (N_0/2)\delta_{nm}$, and $\langle v_n \check{v}_m \rangle = 0$. Then eq. (31) for updating $\hat{\theta}_n$ can be written, after using the linearizing approximation as in (32),

$$\hat{\theta}_{n+1} = (1 - \alpha)\hat{\theta}_n + \alpha\theta_n + \alpha \operatorname{Im}\left(\frac{V_n}{A_n}\right). \tag{35}$$

The random variable $w_n = \operatorname{Im}(V_n/A_n)$ is not gaussian unless $|A_n|$ is constant (pure phase modulation). However, assuming the information symbols and noise are independent, the $\{w_n\}$ are zero-mean and statistically independent with variance $(N_0/2)\langle 1/|A|^2 \rangle$.

By the superposition principle for linear systems, the error in the output of the phase-locked loop is given by

$$\hat{\theta}_n - \theta_n = \operatorname{Re}\{J[F(j\omega) - 1] \exp(j\omega nT)\} + \nu_n, \tag{36}$$

where the sequence $\{\nu_n\}$ satisfies

$$\nu_{n+1} = (1 - \alpha)\nu_n + \alpha w_n, \tag{37}$$

and therefore has zero mean and steady-state variance

$$\lim_{n\to\infty} \langle \nu_n^2 \rangle = \frac{\alpha N_0}{2(2 - \alpha)} \left\langle \frac{1}{|A|^2} \right\rangle. \tag{38}$$

The mean-squared error in the phase estimate is thus

$$\langle (\hat{\theta}_n - \theta_n)^2 \rangle = \frac{|J|^2}{2} |F(j\omega) - 1|^2 + \langle \nu_n^2 \rangle,$$

which from (33) and (38) is

$$\langle (\hat{\theta}_n - \theta_n)^2 \rangle = \frac{|J|^2}{2} \frac{4 \sin^2(\omega T/2)}{\alpha^2 + 4(1 - \alpha)\sin^2(\omega T/2)} + \frac{\alpha N_0}{2(2 - \alpha)} \left\langle \frac{1}{|A|^2} \right\rangle. \tag{39}$$

The residual RMS phase jitter, given by the square root of the above expression, is plotted as a function of the coefficient α for signal-to-noise ratios $\langle |A|^2 \rangle / N_0$ of 30 dB and 22 dB in Figs. 4a and 4b, respectively. In each case, a 16-point QAM constellation is assumed. The higher the phase jitter frequency ω relative to the symbol rate $1/T$, the greater the residual RMS jitter. The curves in Figs. 4a and 4b show the case of no jitter (in which case, the residual jitter results from noise entering the discrete time-phase-locked loop) and also the cases of 14-degree peak-to-peak jitter with $\omega T/2\pi = 1/48$ and with $\omega T/2\pi = 1/20$. The choice of bandwidth of the decision-directed phase-tracking loop, determined by α, should be governed by the highest expected phase jitter frequency. If the spectrum of the phase jitter is known, a higher-order phase-locked loop may permit more effective phase tracking.

For given values of RMS residual phase jitter, the error probability can be approximated as in Ref. 1. For example, we find from Fig. 4b that the residual RMS phase jitter is about 2.5 degrees in the 16-point QAM systems, for $\alpha = 0.3$, when the channel has a signal-to-noise ratio of 22 dB and 14 degrees peak-to-peak channel phase jitter with frequency 1/48 that of the symbol rate. From Fig. 11 of Ref. 1, we find that the resulting error probability is about 4×10^{-7}. The same system

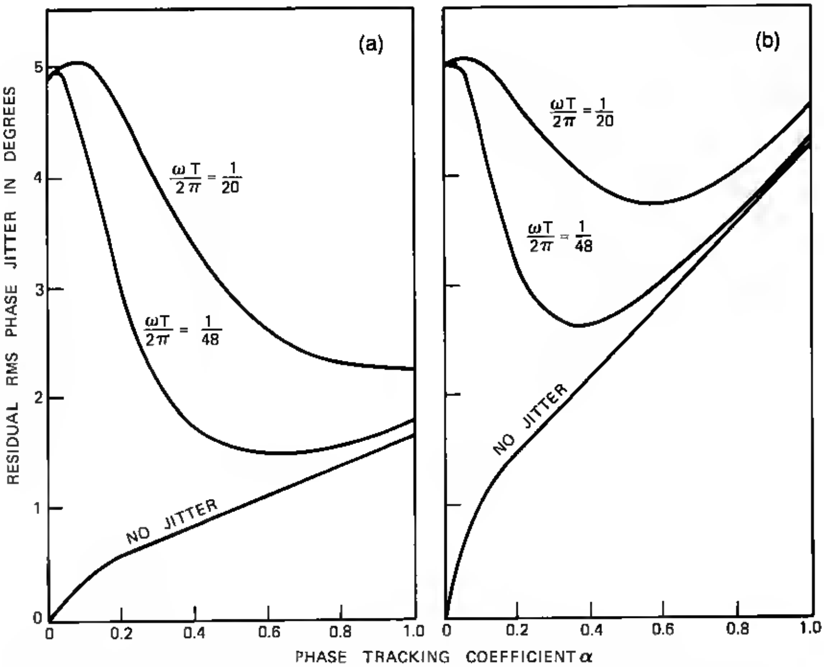

Fig. 4—(a) Residual rms phase jitter for a channel with 30-dB s/n and 14-degree peak-to-peak phase jitter. (b) Residual rms phase jitter for a channel with 22-dB s/n and 14-degree peak-to-peak phase jitter.

with the same value of α in the absence of phase jitter has an error probability of about 5×10^{-8}. The same system with $\alpha = 0$ and no phase jitter bas an error probability of about 10^{-8}.

VI. SIMULATION OF THE QAM RECEIVER

The receiver described in this paper with tbe QAM constellation of Fig. 1 has been simulated on an IBM 370 computer, with 9600-b/s QAM data signals transmitted over real voiceband telephone cbannels as input. The simulation technique and the evaluation of this and other high-speed modems were reported in Ref. 18. In general, over a variety of different voiceband channels, the QAM system's performance appeared to be superior to that of all other systems tested.

One channel used for transmission of the QAM signals consisted of a Holmdel-to-Murray-Hill voiceband channel plus 50-Hz, 17-degree, peak-to-peak sinusoidal phase jitter which was inserted by a line

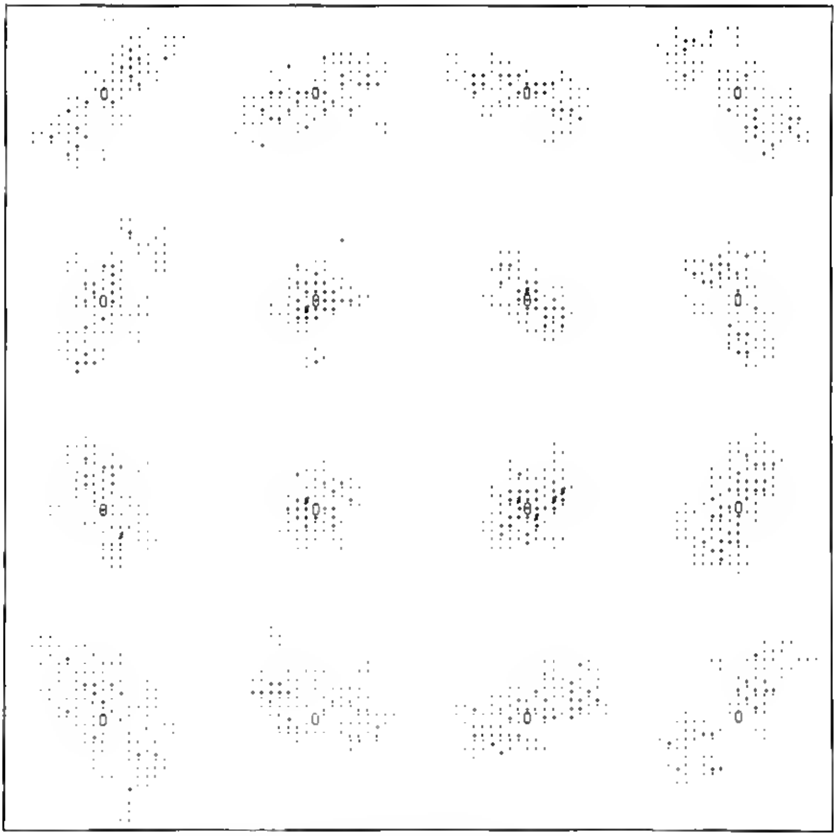

Fig. 5a—Receiver output constellation $\alpha = 0.01$.

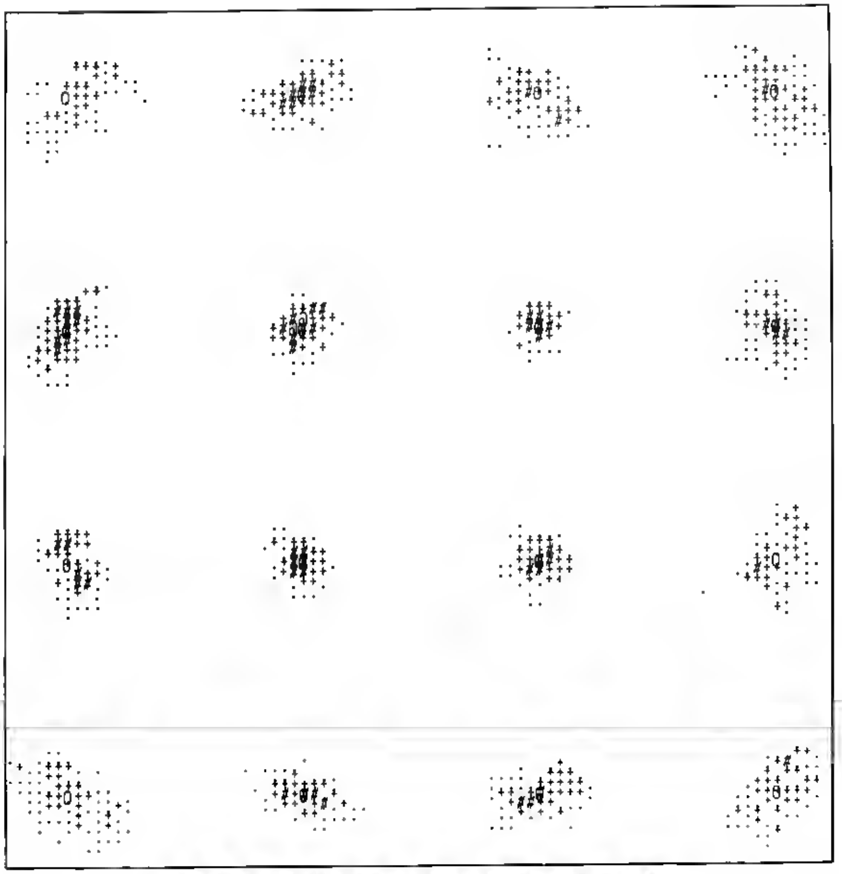

Fig. 5b—Receiver output constellation $\alpha = 0.3$.

simulator. The phase jitter and other impairments contributed by the Holmdel-to-Murray-Hill line alone were not too severe; the worst impairment was second-harmonic distortion, amounting to 32 dB (fundamental to average second-order product).

An illustration of the receiver's effectiveness in tracking and removing sinusoidal jitter from the same recorded data signal is shown in Figs. 5a and 5b, in which the unquantized complex (i.e., two-dimensional) receiver outputs are plotted, $\check{y}_n$ versus y_n. A · indicates that the particular set $(y, \check{y})$ occurred at least once during transmission, a + that it occurred between 4 and 10 times, a # that it occurred between 11 and 20 times, and an @ that it occurred more than 20 times. Thus, these figures are "constellations" or coarsely-quantized two-dimensional histograms of the receiver's demodulated ouput. The coordinates of the possible transmitted information symbols (±1, ±3 for QAM signals) are shown as circles. Figure 5a shows the two-dimensional receiver output constellation for the case when the parameter α is too small to allow the jitter to he tracked; $\alpha = 0.01$. Thus, in this case, the original 17-degree peak-to-peak jitter appeared at the receiver output, resulting in the hanana-like shapes lying along the circumferences of circles centered at the origin. Note that, if the only impairment present was additive random noise, we would expect the scatter plots to look like circles centered on the information symbol coordinates and with radii proportional to the rms value of the noise. Figure 5b is a constellation for the case $\alpha = 0.3$, which allows the sinusoidal jitter to be tracked and almost completely removed by the demodulator.

VII. SUMMARY AND CONCLUSIONS

We have proposed a decision-directed demodulator phase-recovery loop coupled with adaptive passband equalization for use in a two-dimensional, suppressed-carrier, data communications system. Accurate compensation of phase jitter and frequency offset is afforded by placing the demodulator and a sufficiently wide bandwidth decision-directed phase-tracking loop together *following* the equalizer.

The derivation of the receiver's adaptive algorithm for jointly setting the equalizer tap coefficients and the carrier phase estimate was based on a gradient search algorithm for minimizing an expression for the receiver's output mean-squared error. This gradient search algorithm was shown to converge in the absence of noise and phase jitter to a nonunique but optimal set of tap coefficients and carrier phase-angle estimate.

Computer simulations using real-channel received waveforms reported here and in Ref. 18 confirm the feasibility of the QAM receiver structure.

Assuming perfect passband equalization and making a simplifying linear approximation, we analyzed the system's residual phase error as a function of carrier tracking loop gain, signal-to-noise ratio, and the amount and frequency of sinusoidal phase jitter. The optimum value of the carrier-tracking-loop-gain parameter α was seen to depend on the noise and phase-jitter parameters, although reasonable design compromises can be made.

A forthcoming paper[19] will explore the adaptation and tracking hehavior of the comhined equalizer, carrier recovery system, and demodulator in more detail.

The two-dimensional adaptive receiver structure described here can also be extended to systems employing decision feedback equalization. The performance of such a receiver will be reported in a later paper.

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